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AUTOMETERS

1. Description of autometers

The paper is concerned with self – acting measuring instruments. The essence of their operation consists in indicating in the unequivocal manner the final result of measurement on the scale enabling its direct reading, Moreover the measurement process does not require any manual or computational activities besides setting the measured object to the instrument in the proper manner. Taking into account automation of larger and larger number of technological and mechanical processes, making steady routine of production activities and in view of necessity of permanent control of those processes as well as of ready products – the role of such instruments is on the increase.

Let us restrict ourselves to measuring geometrical qualities of objects and in order to better understand the essence of the matter let us make use of the following example. The calculation of the area of a rectangular window pane requires previous measurements of both of its sides and multiplying of the obtained numbers. When it is necessary to repeat those activities several times in spite of it being uncomplicated – the repetitions require quite a lot of time. The whole procedure can be shortened and simplified by using the following instrument. /Fig 1/. To a smooth plate we fasten two perpendicular slips, each of which is equipped with a linear scale. This way the co – ordinate system Oxy has been established. We draw the family of hyperbolas $xy=c$ on the plate. If we put the pane to the plate so that its two edges contact the slips, then the summit of the pane will touch a certain hyperbola $xy=c$. c_0 is the looked for value of the pane area. Writing over each of the families of hyperbolas marked on the plate the corresponding to it value c_0 – we receive the required reading value. That simple instrument has some characteristic qualities. Its scale – on which the result of the measurement is read – is immovable. While making the measurement – the reciprocal position of the measured object and that of the instrument – do not change.

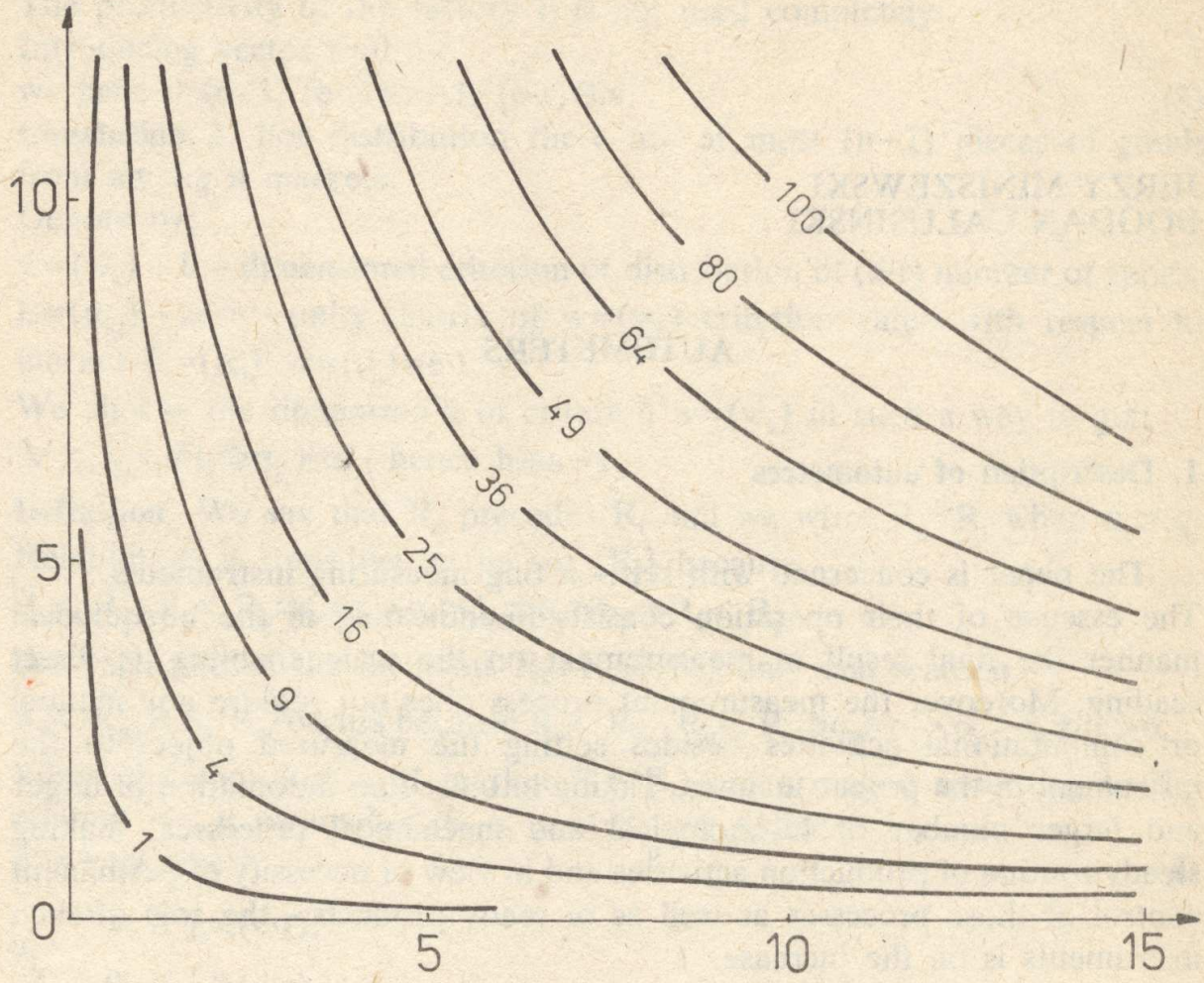


Fig. 1. Autometer to measure the areas of rectangles
Rys. 1. Autometr do mierzenia pól prostąkątów

The value of the measured quality is univocally marked on the scale by the object itself and to express it more accurately – it is marked by its determined in advance characteristic point. The class of the measuring instruments satisfying those conditions will be called nomographical autometers.

Definition 1. We call the nomographical autometer a measuring instrument which satisfies the following conditions:

a) the scale of the instrument does not change its position towards its remaining parts
b) during the measurement the reciprocal position of the object as well as that of the instrument does not change
c) the value of the measured quality marks unequivocally the establis hed in advance point of the measured object of measurement which is easy to place.

If we restrict ourselves to measuring such qualities of objects as areas, parameters and other linear elements of simplest geometrical figures as

well as surface areas, volumes, edge lengths of simplest geometrical solid bodies then objects of measurement by means of the nomographical autometer can be defined as follows:

Definition 2. The object of measurement is such and only such material object whose measured n – quality X

a) is expressed by absolute number of $x_1, x_2, \ldots, x_n$ independent parameters
b) $x_1, x_2, \ldots, x_n$ parameters valies are lengths of segments occurring on the surface of the object and are measurable with a suitable instrument
c) those segments are not set in less than n – dimensional space, they have a common fore – part and angles between them are stable
d) the quality of X value is a function of $x_1, x_2, \ldots, x_n$ parameters, it is a permanent and strictly monotonical one considering each of them.

By autometrisation we mean calculating the value of quality of the object by means of the autometer. In the process either shape or destination of objects is generally not essential. What is only important is the defined analogy of relations among objects and their measured qualities discussed in def. 2.
Thus certain sets of autometrisable objects can be differentiated in identical manner. Let us call them classes of objects being analaguous towards a given quality when autometrisation is concerned.

Definition 3. The class of similar objects – concerning X quality (in the sense of a automation) will further be called the set of all objects satisfying the following conditions

a) X quality of each object is defined by the same set of $x_1, x_2, \ldots, x_n$ independent parameters
b) the value of X quality is defined by the same function of $x_1, x_2, \ldots\ldots, x_n$ $f = f/x_1, x_2, \ldots, x_n$/parameters
c) objects of the same class have respectively equal angles between segments corresponding with $x_1, x_2, \ldots, x_n$ parameters as in def. 2 b, c.

It should be noticed, however, that the definition is not equivalent to the geometrical similarity definition since e.g. all the rectangles are similar in the sense of autometrsation (concerning perimeter or area) though they are geometrically not similar.

The condition of similarity of angles (def .2c) is essential here as it implies impossibility of constructing a nomographical autometer for measurement of lengths which are bent under arbitrary angles and in the arbitrary point of bars. In that case a movable scale is necessary (fig. 2) and in case of non – plane curves a graduated string is, which does not fulfil conditions given in def. 1. The reason for it is an easy conclusion from the following:
Theorem 1 (for nomographical autometrisation):

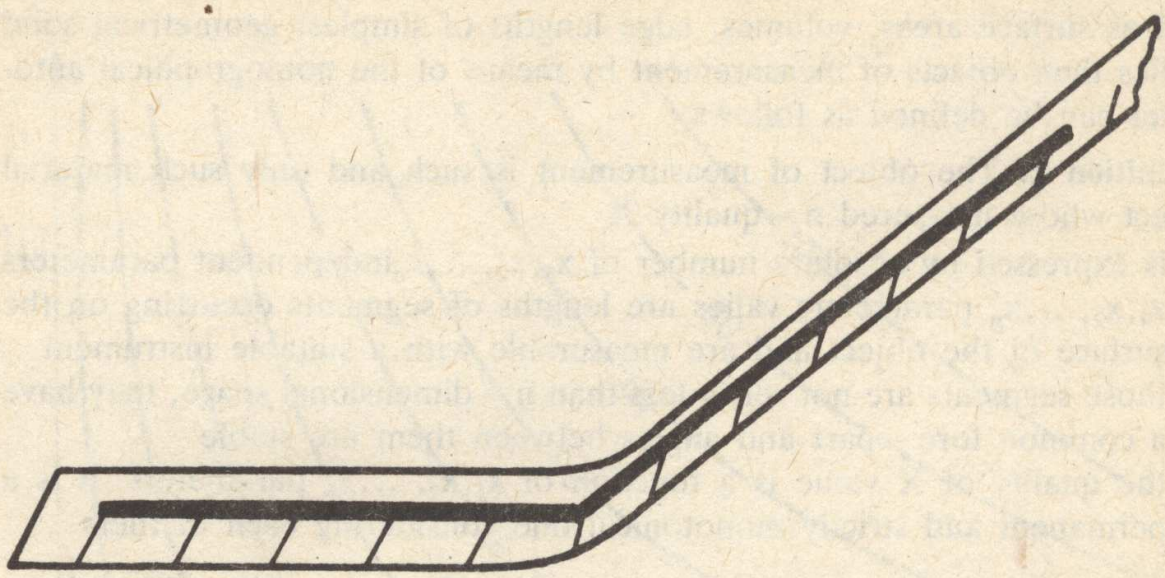


Fig. 2. Autometer to measure lengths of bars bent under an arbitrary anlage
Rys. 2. Autometr do mierzenia długości zgiętych pod dowolnym kątem prętów

If the given X quality a tri – parameteral one at the very most, which value is expressed by the $f=f(x_1,x_2,x_3)$ continous function and is accurately monotonical considering each of the x_1,x_2,x_3 arguments where x_1,x_2,x_3 are independent parameters of X quality then there is a nomographical autometer autometrising each element of the class of similar objects when taking into consideration the x quality.

Proof *)

From def. 2 segments corresponding with x_1,x_2,x_3 parameters of x quality extend from one point and are not situated in one plane. Thus they determine the co – ordinate system in space, in which units on the axes are equal to units of adequate parameters. In that system the $f(x_1,x_2,x_3)=c$ equation presents a continous surface and each point of that surface determines the same value of X quality. By changing c we get the surface family having the same quality. If we set the object so that the segments charakterising it should be paralled to the corresponding with the system axes then their common point would be on one of the family surfaces which thus determines X quality value. The co – ordinate system with its $f(x_1,x_2,x_3)=c$ family is a nomographical autometer for all the similar objects when considering X quality as it fulfils the conditions of definition 1. To autometrise an object it is enough to place it in the above – mentioned way and to read the quality value on the indicated surface.

The ruler is the simplest nomographical autometer as it is points aut the one parameter value of the object – its length ar width. The autometer

*) To simplify we give a proof for measured objects with tri – parameter qualities here. For bi – and one parameter qualities the reasoning course is similar.

shown in (fig. 3) points out the one – parameter value of the object likewise. To a smooth plate two perpendicular slips are fixed. One of them has a

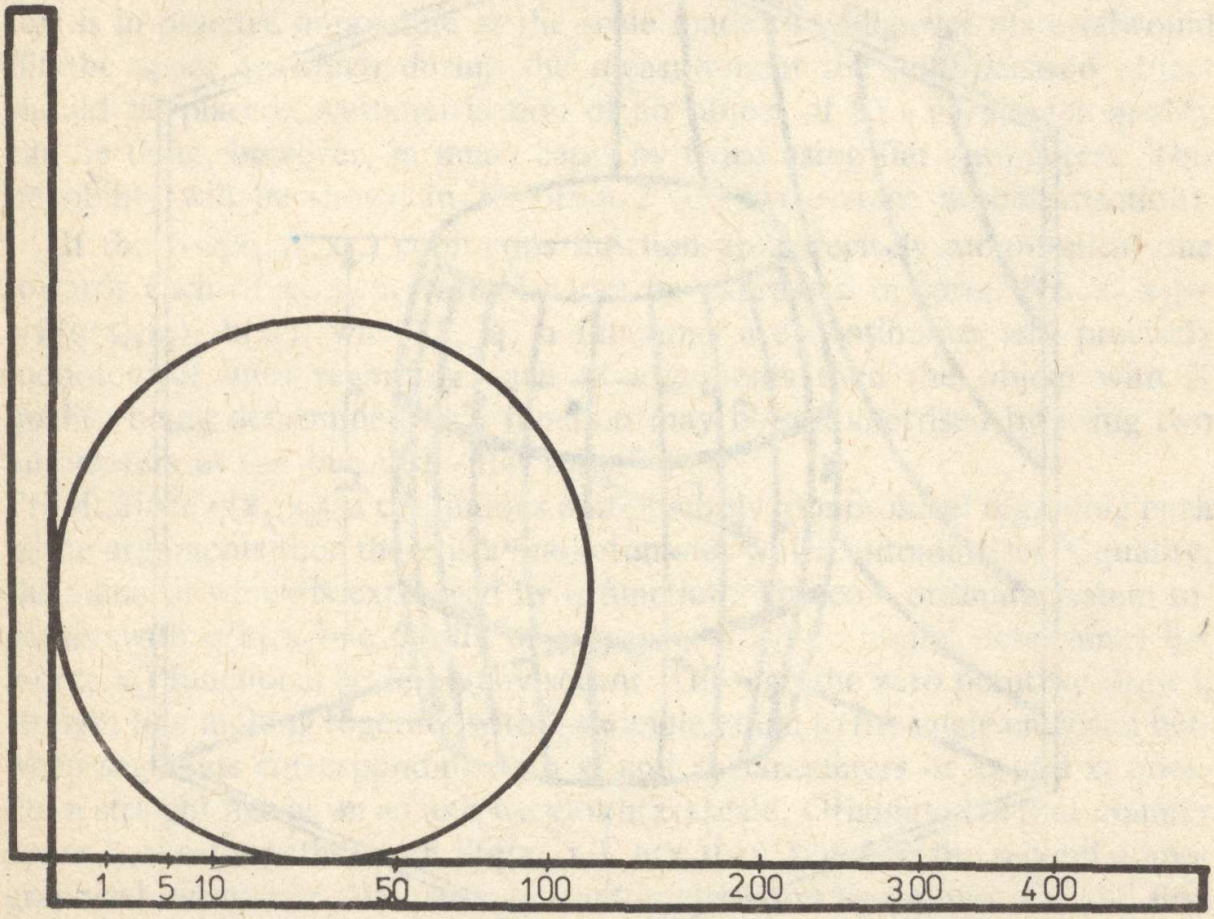


Fig. 3. Autometer to measure circle areas
Rys. 3. Autometr do mierzenia pól kół

square scale $\xi=\pi r^2$. The circle is placed contiguously to both slips. The point of tangency with the scale will show the value of the area. The instrument serves the tinsmith to evaluate the area of the used sheet iron block which is sometimes necessary to price strictly the service performed by him (We mean to estimate it and not to measure it as evaluation at a guess of the point of tangency of the straight line and of the circle is in general insufficiently precise). It may be done in a much more exact manner e.g with the autometer shown in fig. 4. The same autometer after replacing the square scale with the adequate linear one will serve of the circumference, diameter or radius measurements (fig. 4). The principle of its structure may easily be applied to autometrisation of spheres volume, their surface areas, diameters and to other linear elements. The same regards as well respective classes of similar polygons or ones of similar solid bodies Leonard Euler searched for a simple instrument to measure the volume of barrels in the brewery. The requirements of generally

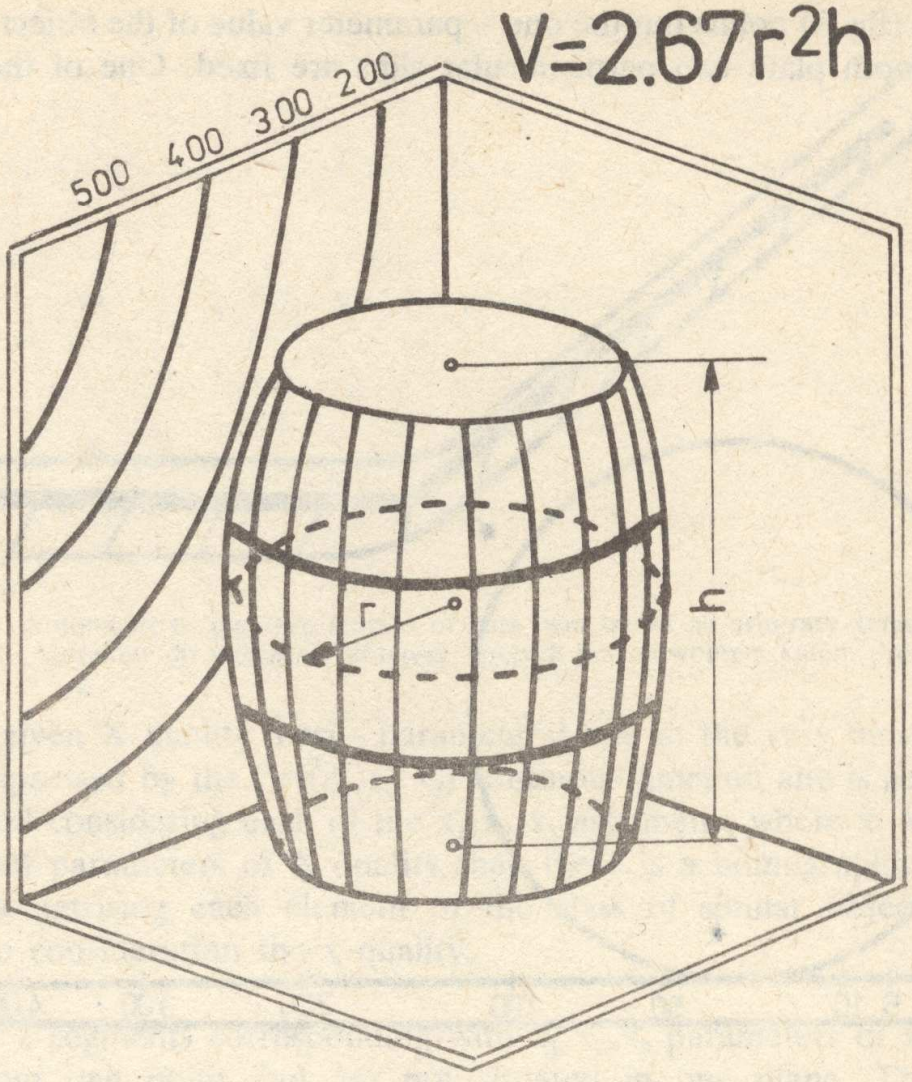


Fig. 4. Autometer to measure barrel volumes
Rys. 4. Autometr do mierzenia pojemności beczek

understood simplicity are evidently fulfilled by nomographical autometers. In that case one may make use of such an instrument as that shown in fig.4. It is assumed here that the barrel is a solid body originated from a rotary ellipsoid with r,r,l semi – axes by cutting both bowls off in the distance of $\frac{2}{3}$l from the mid – point. Then the volume of the barrel is expressed with sufficient precision by $v=2{,}67r^2h$ formula, where h is the height of the barrel *).

Nomographical autometers serving to point out one – parameter qualities of objects will be called linear autometers and those two – parameter ones – flat autometers. Thus the ruler and the autometer shown in fig. 3 are linear autometers and those shown in fig.1 and 4 are flat ones. Autometers

*) One can obtain more precise results by approaching the stave are with the parabola arc. For theoretical considerations regarding autometrisation it is of no great importance, however.

serving to measure qualities of tri – parameter objects would respectively be called spatial ones. The structure of those spatial nomgraphical autometers is in pracitce impossible as the scale made of whichever materialwould fill the space in which during the measurement the autometrised object should be placed. Autometrisation of an object of tri – parameter quality can be done, however, in many cases by twice using flat autometers. This possibility will be shown in Theorem 2 (on two – stage autometrisation).

If the $f=f(x_1,x_2,x_3,)$ coninuous function and precisely monotonical one towards each of x_1,x_2,x_3 variables can be expressed in form: $f(x_1,x_2,x_3)=$ $=F[q(x_1,x_2), h(x_3)]$ when F, q, h functions are continuous and precisely monotonical ones regarding each of arguments then the object with X quality being determined by f function may be autometrised by using two autometers at the outmost – flat ones.

Proof: Since $q(x_1,x_2)$ is continuous and precisely monotonical regarding each of the arguments then there is a flat autometer which autometrises X quality, the value of which is expressed by q function. The co – ordinate system together with $q(x_1,x_2)=c$ family of curves $q(x_1,x_2)=c$ family determines $\xi=$ $=q(x_1,x_2)$ functional scale on any secant. Through the zero point we draw η straight line making together with ξ an angle equal to the angle enclosed between segments corresponding with x_1 and x_3 parameters or x_2 and x_3 ones. On η straight line as on an axis we plot $h(x_3)$ scale. Originated in that manner a new ξ, η set together with $F[q(x_1,x_2), h(x_3)]=C$ family is the second nomographical autometer. We carry out autometrisation as follows. On the first autometer we place the object suitably with x_1,x_2 surface and then the common point of segments indicates a certain curve of $q(x_1,x_2)=c$ family. Next on the second autometer we place the object suitably with x_1, x_3(or x_2,x_3) surface the way that the segment corresponding with x_3 should be parallel to η and its end should be placed in the intersection point of $q(x_1,x_2)=c$ curve with axis. The common end of segments indicates the sought value of X quality.

The method of measurement presented in theorem 2 may be generalized on any continuous functions of $f(x_1,x_2,x_3)=F_1[q_1(x_1,x_2), h_1(x_3)]+F_2[q_2(x_1,x_2),$ $h_2(x_3)]$ forms for q_1, q_2, h_1, h_2 continuous and strictly monotonical ones regarding each of the variables by discovering $F_1[q_1(x_1,x_2), h_1(x_3)]=c_1$, and $F_2[q_2(x_1,x_2), h_2(x_3)]=c_2$ families of curves and subsequently constructing from two secants a new co – ordinate system and discovering in it $F_1+F_2=c$ family. In that case, however, the measurement will not be autometrisation since the quality value will not be indicated by the object subjected to measurement. That generalization may inductively be extanded for any number of $F_i[q_i(x_1,x_2), h_i(x_3)]$ components where $i=1,2,\ldots,n$. In patricular one can measure objects, the qualities of which are exppressed by any polynominal with three variables. Fig. 5–7 present examples of various nomographical autometers. Their descriptions contain nature and application of those instruments.

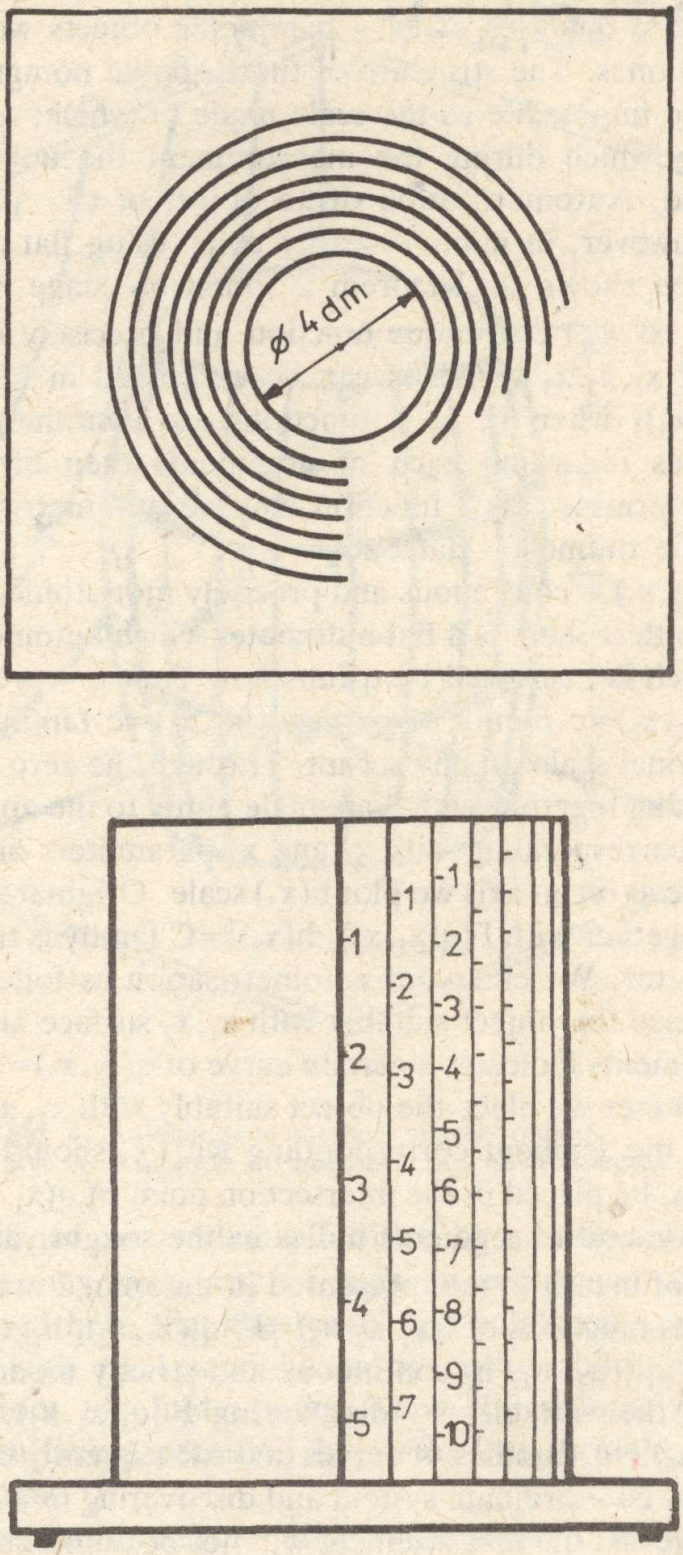


Fig. 5. Autometer to measure open and emptycan volumes
a) top view
b) front view

Rys. 5. Autometr do mierzenia pojemności otwartych puszek
a) widok z góry
b) widok z przodu

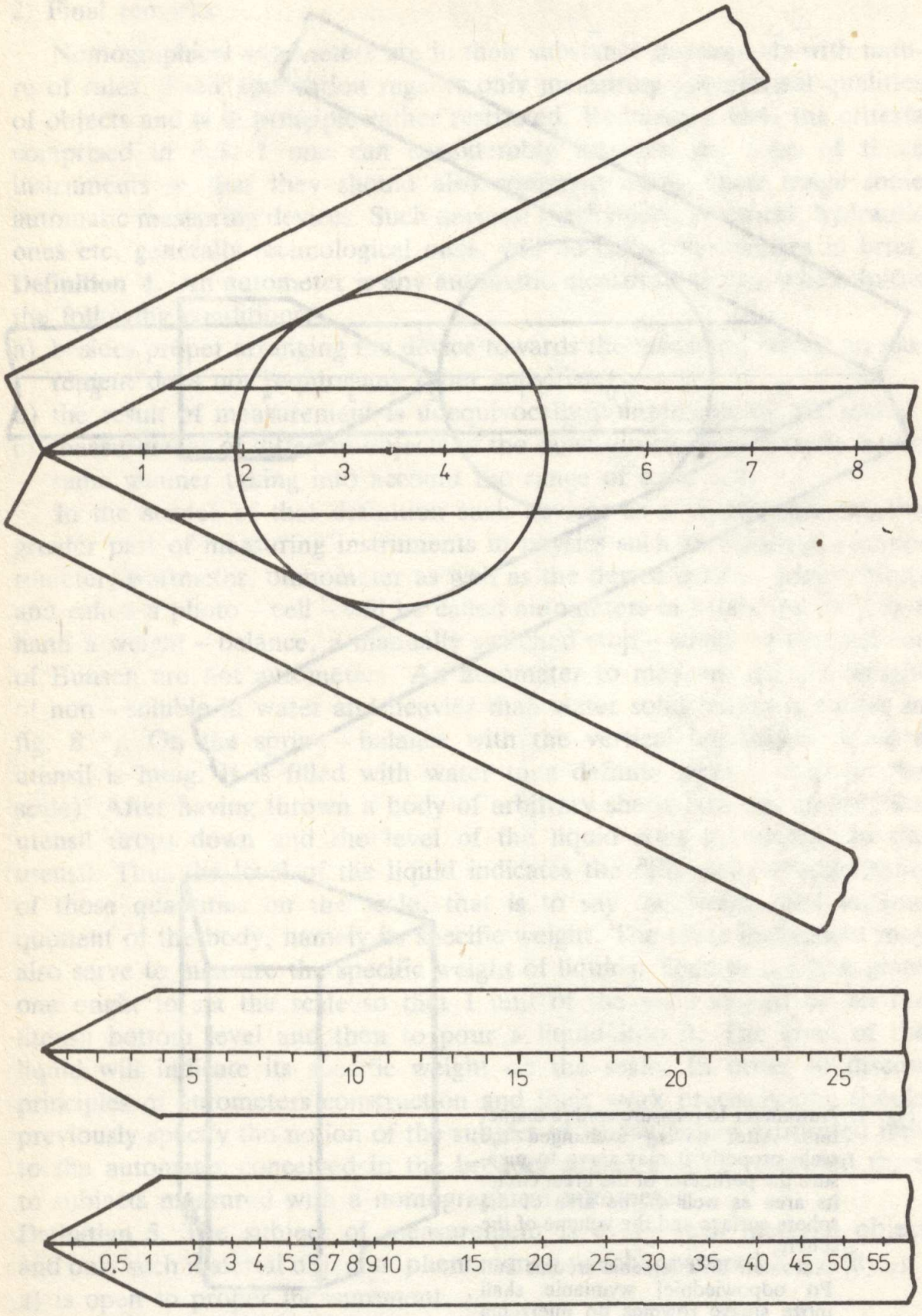
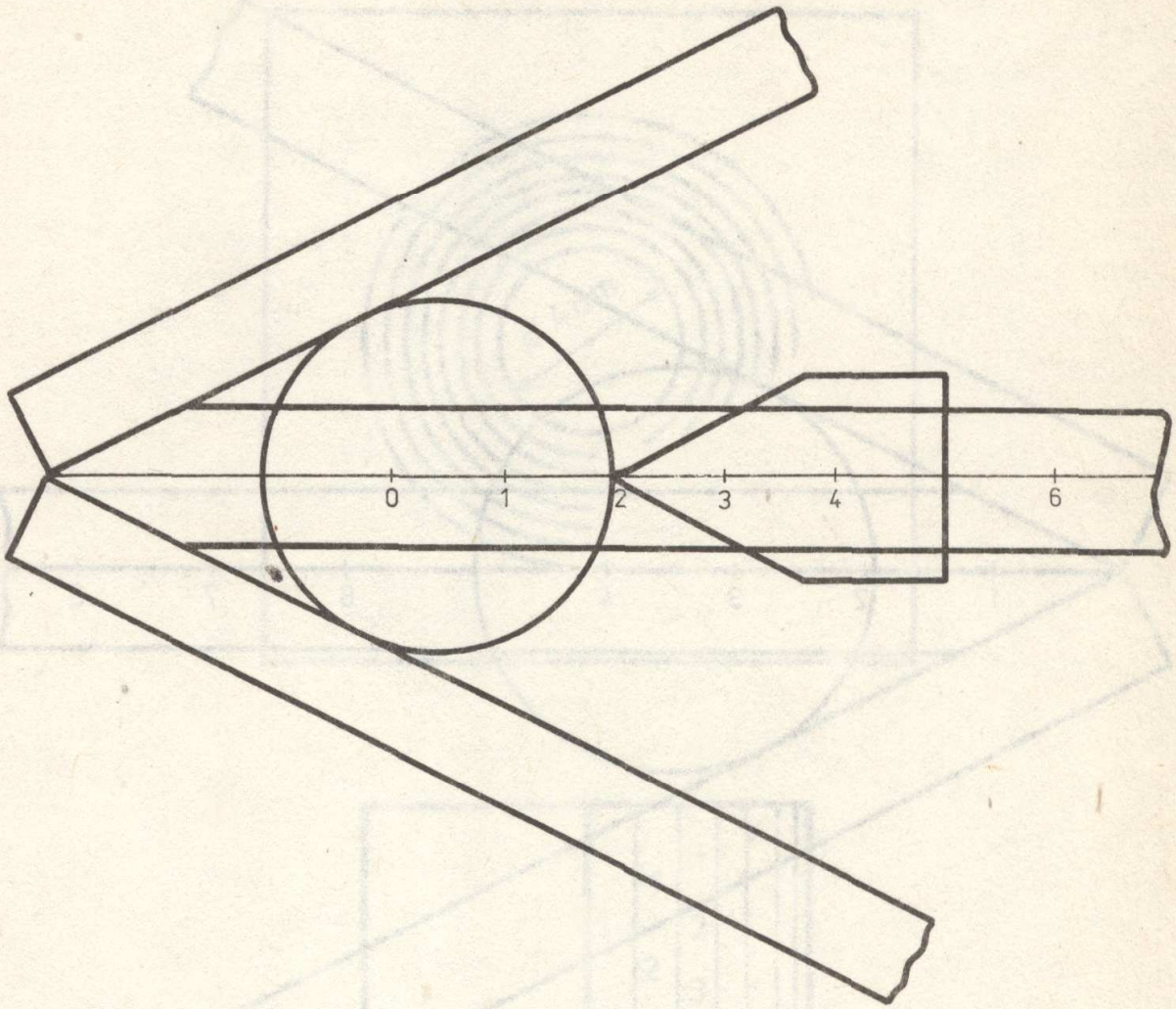


Fig. 6. Autometer to measure diameters, perimeters and areas of circle
Rys. 6. Autometr do mierzenia średnic, obwodów i pól kół



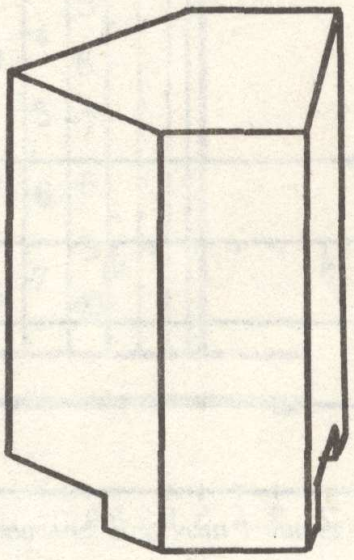


Fig. 7. Autometer to measure sphere diameters. After having exchanged the scale properly it may serve to measure the perimeter of the great circle, its area as well as the area of the sphere surface and the volume of the sphere

Rys. 7. Autometr do mierzenia średnic kul. Po odpowiedniej wymianie skali może służyć również do mierzenia obwodu koła wielkiego kuli, jego pola, pola powierzchni kuli i jej objętości

2. Final remarks

Nomographical autometers are in their substance instruments with nature of rules. Their application regadrs only measuring geometrical qualities of objects and is in principle rather restricted. Reducing a little the criteria comprised in def. 1 one can considerably broaden the class of those instruments so that they should also comprise within their range some automatic measuring devices. Such devices: mechanical, electrical, hydraulic ones etc, generally technological ones, will be called autometres in brief.

Definition 4. An autometer is any automatic measuring device which fulfils the following conditions:

a) besides proper arranging the device towards the measured object measurement does not require any other activities,
b) the result of measurement is unequivocally pointed out on the scale,
c) measurement of different objects of the same dimension are made in the same manner taking into account the range of scale only

In the seance of that definition such devices as a tangentbalance, the greater part of measuring instruments in physics such as voltmeter, amperometer, wattmeter, ohmometer as well as the device used in competitions and called a photo – cell – will be called autometers in brief. On the other hand a weight – balance, a manually switched stop – watch or photometer of Bunsen are not autometers. An autometer to measure specific weight of non – soluble in water and heavier than water solid bodies is shown in fig. 8 *). On the spring – balance with the vertical logarithmic scale a utensil is hung. It is filled with water to a definite level (1 unit on the scale). After having thrown a body of arbitrary shape into the utensil, the utensil drops down and the level of the liquid rises in relation to the utensil. Thus the level of the liquid indicates the difference of logarithms of those quantities on the scale, that is to say the weight and volume quotient of the body, namely its specific weight. The same instrument may also serve to measure the specific weight of liquids. Then in the first place one ought to set the scale so that 1 unit of the scale should be on the utensil bottom level and then to pour a liquid into it. The level of the liquid will indicate its specific weight on the scale. In order to discuss principles of autometers construction and their work precisely one should previously specify the notion of the subject of measurement attributed now to the autometer conceived in the broader sense. Def. 2 restricted them to subjects measured with a nomographical autometer.

Definition 5. The subject of measurement is every such material object and only such material object or phenomenon which measured X quality:

a) is open to proper measurement

*) Water may naturally be replaced by another liquid.

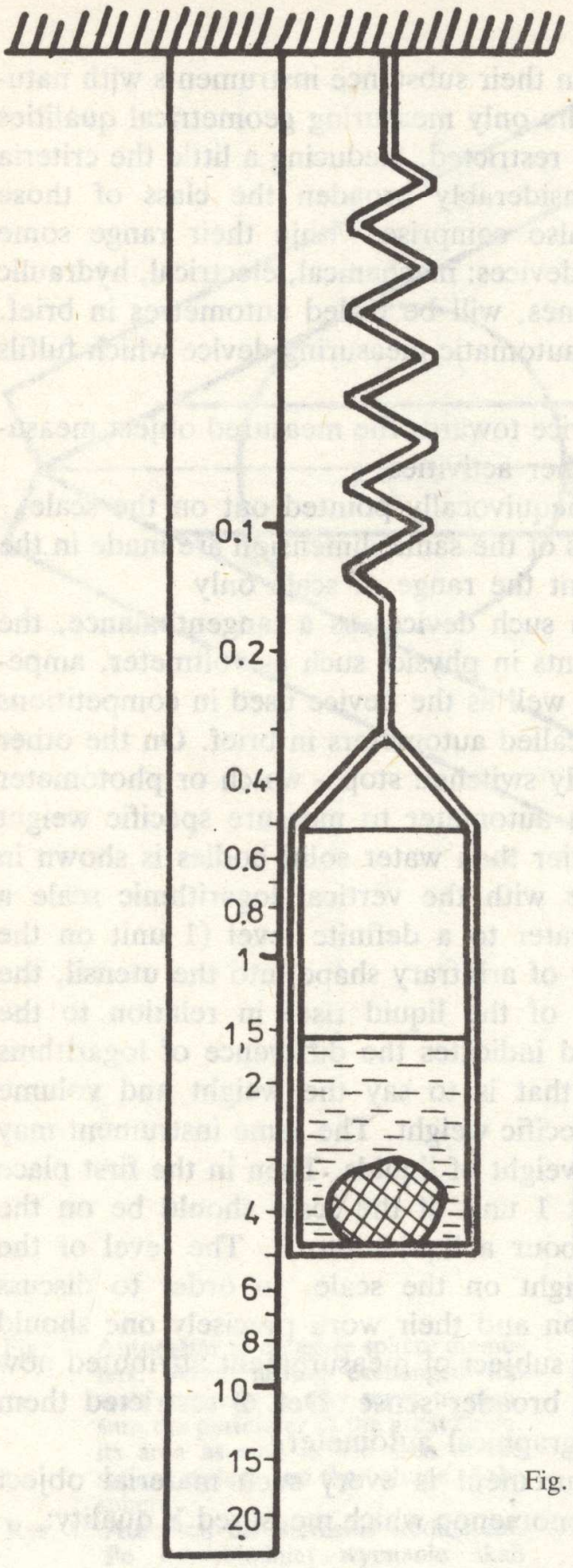


Fig. 8. Autometer to measure the specific weight of liquids as well as the specific weight of solid bodies, non – soluble ones in a liquid

Rys. 8. Autometr do mierzenia ciężaru właściwego cieczy oraz ciał stałych nierozpuszczalnych w cieczy

b) is expressed by a finite number of $x_1, x_2, \ldots, x_n$ independent parameters,
c) X quality value is a continuous function of $x_1, x_2, \ldots, x_n$ variables and a precisely monotonical one taking into account each of them in the scope of a carried out measurement.

Such comperehension of a subject of measurement and of an autometer greatly enlarges the range of objects and of autometrisable phenomena as well as the scope of measuring devices. Let us investigate that problem more thoroughly:

a) autometrisation does not only concern material objects but also physical phenomena such as electric current flow,attraction, pressure, temperature,
b) resigning from requirements included in conditions a and c of def .1 makes possible applying in autometers such mechanical devices as levers, transmission, gear etc. It permits to use various solutions both theoretical and technological ones.

It creates great inventive possibilities

c) in case when direct autometrisation of a given quality is impossible when it is based on a generally accepted mathematical formula it enables to investigate indirect methods consisting in using physical properties of bodies and their relevence to mathematical formulas.

Here let us make use of the following example illustrating well that problem: as it results from our considerations in the first part of this article, it is not possible to construct a nomographical autometer to measure cubicoidal block volumes, e.g made of heavy metal. It results from the fact that V block volume is $V=xyz$ tri – parameter quality where x,y,z indicate lengths. The fact of the matter is that it is imposible to construct a parameter serving that purpose and one bulit on the given formula. It is possible, however, to change the formula and replace it with physical operations. Thanks to it a properly graduated burette being filled with a liquid will become the searched for autometer.

d) Thanks fig. 2 to the possibility of introducing into the device additional elements of the indicated set the preciseness of reading may be increased and even be able to note down indications automatically.

Definition 6. So by a class of measured subjects, similar ones regarding a given value we will mean here a set of all objects of measurement, autometrisation of which is carried out with the same instrument and follows the same course. It should be noticed that definitions 4, 5 and 6 have orginated from definitions 1, 2 and 3 respectively by weakening conditions occuring in them. It results therefore that each nomographical autometer is an autometer in broader understanding and each material subject constituting a subject of nomographical autometrisation is simultaneously a metrisation subject in the sense of def. 5. The same remark regards classes of measured objects concerning a given quality.

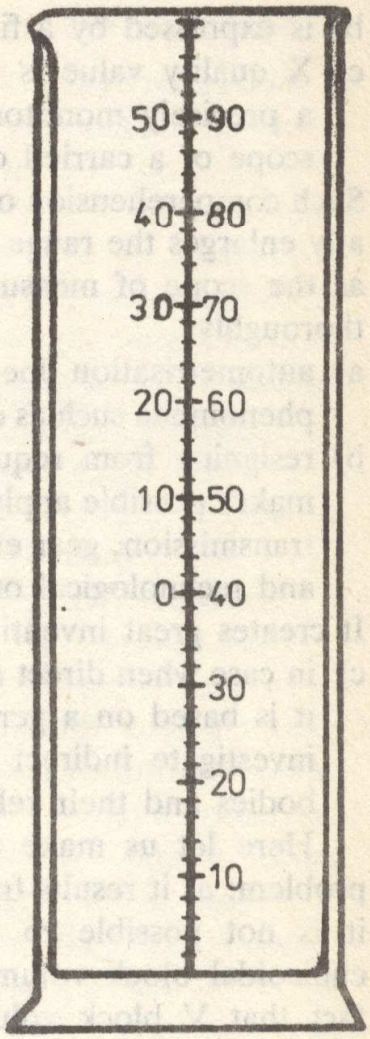


Fig. 9. Burette – autometer to measure the liquid volume and the volume of solid bodies, non – soluble ones in a liquid
Rys. 9. Mensurka – autometr do mierzenia objętości cieczy oraz ciał stałych nierozpuszczalnych w cieczy

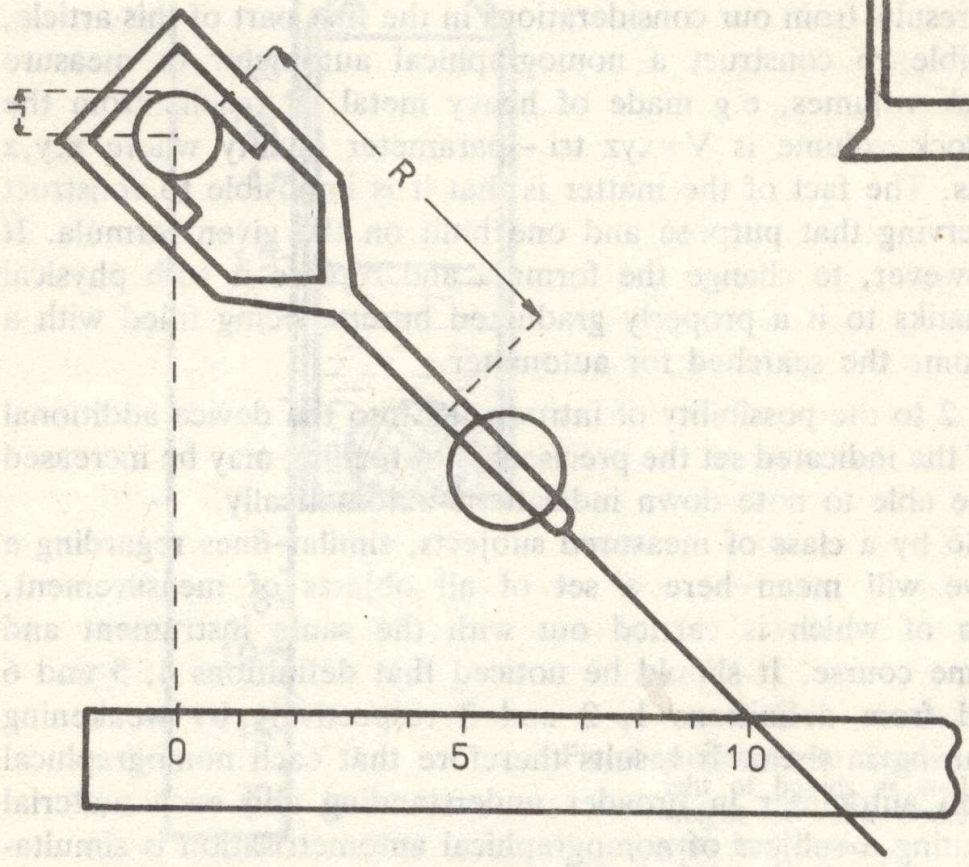


Fig. 10. Autometer to measure the coefficient of friction
Rys. 10. Autometr do mierzenia współczynnika tarcia

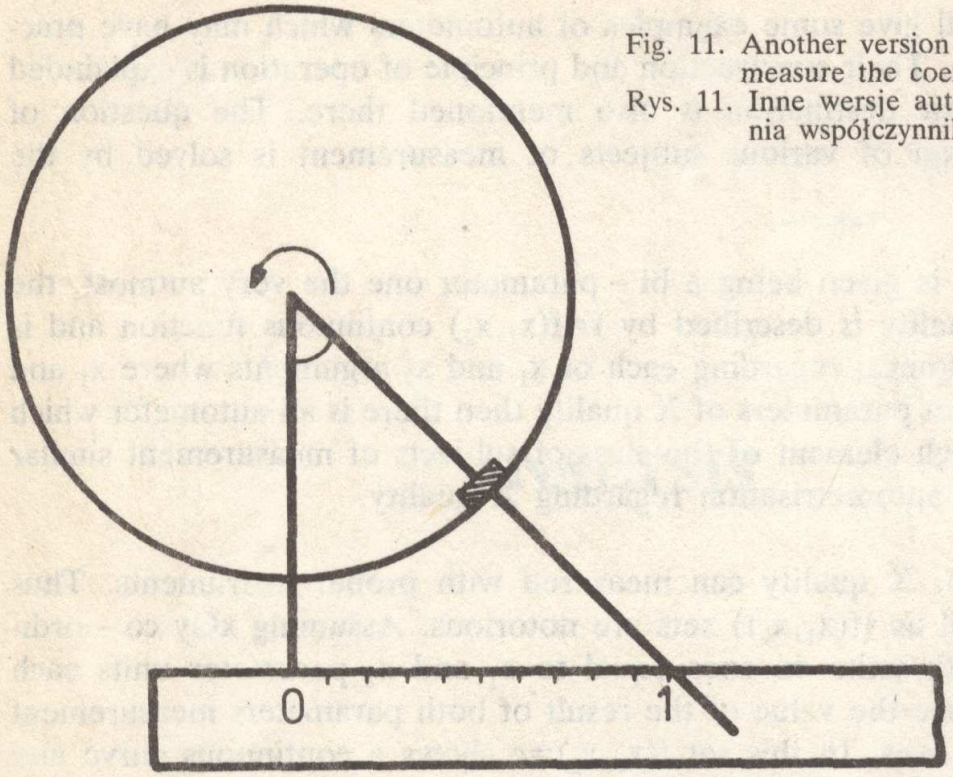


Fig. 11. Another version of the automer to measure the coefficient of friction
Rys. 11. Inne wersje autometru do mierzenia współczynnika tarcia

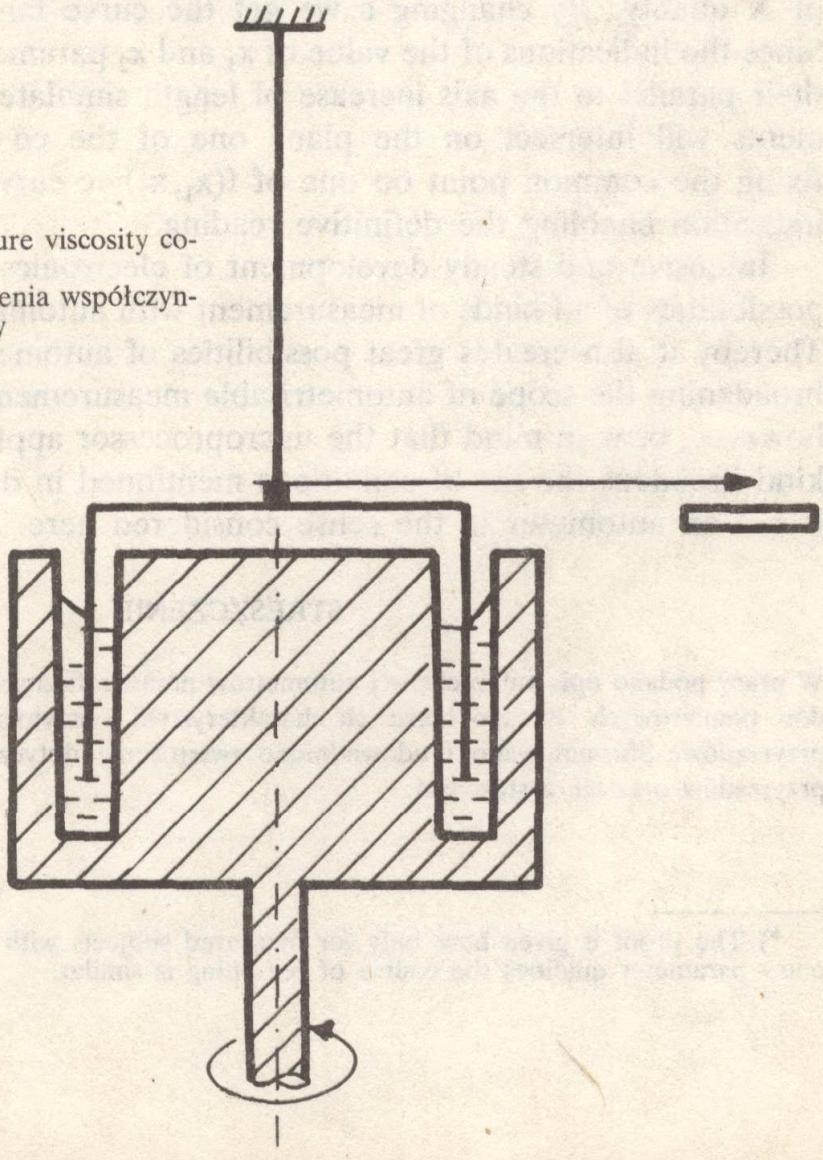


Fig. 12. Autometer to measure viscosity coefficient of liquids
Rys. 12. Autometr do mierzenia współczynnika lepkości cieczy

Here we will give some examples of autometers which may have practical application. Their construction and principle of operation is explainded in fig. 10. Their destination is also mentioned there. The question of auto – metrisation of various subjects of measurement is solved by the following:

Theorem 3

If X quality is given being a bi – parameter one the very autmost the value of the quality is described by $f=f(x_1,x_2)$ continuous function and is precisely monotonical regarding each of x_1 and x_2 arguments where x_1 and x_2 are independet parameters of X quality then there is an autometer which autometrises each element of the class of subjects of measurement similar in the sense of autometrisation regarding X quality.

Proof *)

From def. 5. X quality can measured with proper instruments. Thus $\{(x_1,x_2)\}$ as well as $\{f(x_1,x_2)\}$ sets are notorious. Assuming xOy co – ordinate system with units on axes equal to x_1 and x_2 parameter units each time we can place the value of the result of both parameters measurement on each of set axes. In this set $f(x_1,x_2)=c$ shows a continuous curve and at the same time each point of that curve corresponds with the same value of X quality. By changing c we get the curve family with this property. Since the indications of the value of x_1 and x_2 parameters are simultaneous – their parallel to the axis increase of length simulated by pointers of instruments will intersect on the plane one of the co – ordinate system thus fixing the common point on one of $f(x_1,x_2)=c$ curve family. It is the final indication enabling the definitive reading.

Intensive and steady development of electronics constantly creates new possibilities of all kinds of measurement with automatic elaboration of data. Thereby it also creates great possibilities of autometer construction and of broadening the scope of autometrisable measurement subjects. One should, however, bear in mind that the microprocessor applied in many sets of the kind broadens the list of conditions mentioned in def. 4. Thus such device is not an autometer in the sense considered here.

STRESZCZENIE

W pracy podano opis autometrów i autometrów nomograficznych jako pewnej klasy przyrządów pomiarowych. Podano także ich charakterystyki i przytoczono różne przykłady takich przyrządów. Sformułowano i udowodniono twierdzenie dotyczące konstruowalności takich przyrządów oraz ich zastosowań.

*) The proof is given here only for measured subjects with bi – parameter qualites. For one – parameter qualities the course of reasoning is similar.