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TRISEQUENTIAL VERSION OF THE BOČVAR AND HALLDEN LOGICS

Introduction

System of the three - valued logics was constructed for various purposes, for exemple to the analysis of logical antinomies (see, e. g. [2], [3]), partial recursive functions (cf [8]), and three - valued logic (with true, false and nonsense sentences). Another application includes the investigation of the logic of programs (cf[7]).

It is assumed that logical values are true, false and nonsense or undefined value. Let $V=\{p_i : i \in N\}$ be a denumerable set of propositional variables, $\{\sim, T, \neg, \wedge\}$ and $\{\sim, S, \wedge\}$ be logical connective of the Bocvar and Hallden logic respectively. The connective $\sim, T, \neg$, and S are one - argument and $\wedge$ is the two - argument connective. The set of formulae is defined in the usual way (cf H. Rasiowa and R. Sikorski [10]).

The presented trisequential version is, in a way , dual representation to D. Bočvar and B. Finn [2], O. Anshakov [1] and S. Surma [12].

1. Sequents. Let X, Y, Z be finite sets particulary empty of formulas. A sequent is an ordered triple

$$X+Y+Z$$

of sets of formulas. The sequents will be denoted by Π, Σ, with indices if nesessary.
By an overfilled sequents we mean a sequents $\Sigma = X+Y+Z+$ such that $X \cap Y \neq 0$ or $X \cap Z \neq 0$ or $Y \cap Z \neq 0$.

The presented version of the Bočvar and Hallden calculus consists of rules and sequents, particulary, overfilled sequents.

2. Rules. Rules of the introduction for connectives to sets X, Y, and Z of the sequent X+Y+Z in Bočvar logic will have the following form:

(a) For the connective

$$(R_1^{\sim})\frac{X+Y+Z,\alpha}{X,\sim\alpha+Y+Z},(R_2^{\sim})\frac{X+Y,\alpha+Z}{X+Y,\sim\alpha+Z},(R_3^{\sim})\frac{X,\alpha+Y+Z}{X+Y+Z,\sim\alpha}$$

here X, α mean $X\cup\{\alpha\}$

(b) For the connective T

$$(R_1^T)\frac{X,\alpha+Y+Z;X+Y,d+Z}{X,T\alpha+Y+Z},(R_3^T)\frac{X+Y+Z+,\alpha}{X+Y+Z,T\alpha}$$

(c) For the connective ¬

$$(R_1^{\neg})\frac{X+Y,\alpha+Z;X+Y+Z,\alpha}{X,\neg\alpha+Y+Z},(R_3^{\neg})\frac{X,\alpha+Y+Z}{X+Y+Z,\neg\alpha}$$

(d) For the connective ∧

$$(R_1^{\wedge})\frac{X,\alpha,\beta+Y+Z;X,\alpha+Y+Z,\beta:X,\beta+Y+Z,\alpha}{X,\alpha\wedge\beta+Y+Z}$$

$$(R_2^{\wedge})\frac{X,\alpha+Y,\beta+Z;X,\beta+Y,\alpha+Z;X+Y,\alpha,\beta+Z;X+Y,\alpha+Z,\beta;X+Y,\beta+Z,\alpha}{X+Y,\alpha\wedge\beta+Z}$$

$$(R_3^{\wedge})\frac{X+Y+Z,\alpha,\beta}{X+Y+Z,\alpha\wedge\beta}$$

Rules of the introduction for connectives to sets X, Y and Z of the sequent X+Y+Z in Hallden logic are the same as in Bočvar logic except rules for the connective S. (The connective T in Hallden logic does not occur). For the connective S rules will be in the following form

$$(R_1^S)\frac{X+Y,\alpha+Z}{X,S\alpha+Y+Z},(R_3^S)\frac{X,\alpha+Y+Z;X+Y+Z,\alpha}{X+Y+Z,S\alpha}$$

3. Theorems. We shall call a system $D=(P,P',x_o,R)$ a proof tree if:

(i) P is a finite set, the system (P,R,x_o) is the lower semilattice with the zero element x_o,

(ii) $P'=\{y:\{x:xRy\}=0\}$

(iii) the cardinality of the set $\{x:xRy\}$ is at most 3^2

The sequent $\Sigma = X_1 + X_2 + X_3$ is the terminal sequent if there exists a formula α and $j, 1 \leqslant j \leqslant 3$ so that for every $i, 1 \leqslant i \leqslant 3, i \neq j, S_i = 0$ and $S_j = \{\alpha\}$.

The terminal sequent Σ has a proof on the ground of the set of sequents Φ if and only if there exists a proof tree $D = (\Delta, \Phi, \Sigma, R)$, where $\Phi \subseteq \Delta$ and the relation

$$R = \cup\{R_1^{\sim}, R_2^{\sim}, R_3^{+}, R_1^{T}, R_3^{T}, R_1^{\neg}, R_3^{\neg}, R_1^{\wedge}, R_2^{\wedge}, R_3^{\wedge}\}$$

in the Bočvar logic and

$$R = \cup\{R_1^{\sim}, R_2^{\sim}, R_3^{\sim}, R_1^{S}, R_3^{S}, R_1^{\wedge}, R_2^{\wedge}, R_3^{\wedge}\}$$

in the Hallden logic.

A propositional formula α is a theorem in the trisequential calculus if and only if there exist the overfilled sequents on the ground of which the following terminal sequents are proved:

$$\alpha + 0 + 0$$

$$0 + \alpha + 0$$

in the case of the Bočvar logic and the terminal sequent

$$\alpha + 0 + 0$$

in the Hallden Logic.

The trisequential calculus, presented above permit to choose from the set formulas not only true formulas, but also false and undefined formulas.

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STRESZCZENIE

Celem tej pracy jest przedstawienie trójsekwentowej wersji dwóch systemów rachunku zdań znanych w literaturze jako logiki nonsensu.