Prace Naukowe Wyższej Szkoły Pedagogicznej w Częstochowie
Matematyka VIII, Częstochowa 2000 - 2001

Some Peculiarities and Qualities of Queueing Systems with Random Volume Demands

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Queueing theory is actually used for information systems characteristics determination. It is a division of probability theory, it's models are named queues or queueing systems. Basic random variables analysed in the theory are:

1) number of demands presenting in the system (on service or wariting for service);
2) waiting time and sojourn time of demands in the system;
3) probability of demand losses (which are possible due to different restrictions);
4) probability of different exceeds of certain borders, which are known (for example, probability, that waiting time will be more than T).

Let us assume, that every demand may be characterised by some random volume ζ, which does not depend on volumes of other demands, nor on moments, in which demands come to the system. Generally service time ξ of the demand depends on it's volume ζ only. The joint distribution function of ζ and ξ random variables is

$$F(x,t) = \mathbf{P}\{\zeta < x, \xi < t\}.$$

It's clear, that above assumptions permit to production of new division in queueing theory which may be used for buffer space determination when information system designing. Note, that such problems can't be generally solved by means of classical queueing theory.

For solving the problem we have introduce $\sigma(t)$ random process, where $\sigma(t)$ is a total sum of demands presenting in the system volumes. This process was named summarized volume. It's clear, that in some queuing models $\sigma(t)$ process may be restricted by some constant value $V > 0$, which is named memory volume [3]. Then losses of demands may be possible due to such restriction. Obviously, that these losses differ from ones taking place due to restrictions of waiting places in queues.

On the other hand the dependence between ζ and ξ random variables does solving the problem of determination of $\sigma(t)$ process characteristics not trivial even in the case of $V = \infty$.

Now let us define main classes of problems connected with random volume demands queueing systems.

The general problem is illustrated by fig. 1.

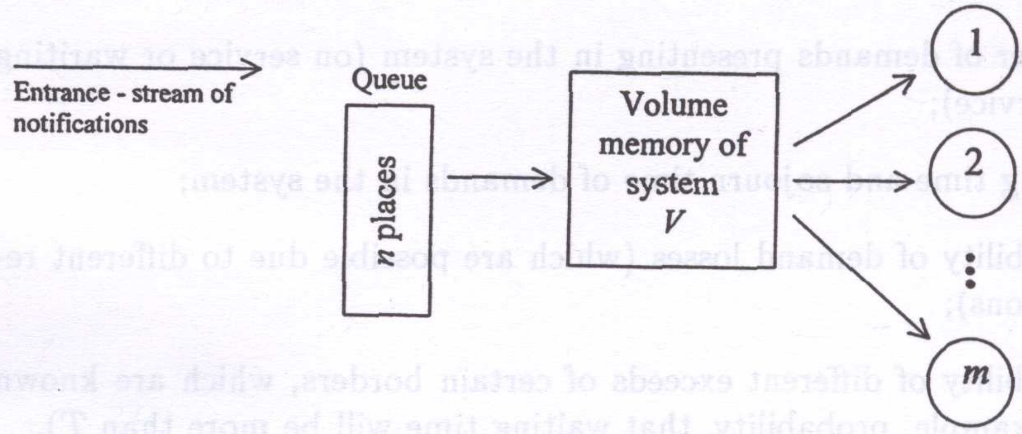

Fig.1

Every demand coming to the system may be lost at the time moment τ of it's coming, if

1) there is no place in the queue at this time moment;

2) the volume x of the demand is such, that $x + \sigma(\tau^-) > V$ (in the case of $V < \infty$).

Now we may represent the next classes of queueing models with random volume demands.

1. Models of queues with districted memory volume ($V < \infty$). For such queues solution of the problem is possible when random variables ζ and ξ are independent ($F(x,t) = L(x)B(t)$, where $L(x) =$

$= \mathbf{P}\{\zeta < x\}$, $B(t) = \mathbf{P}\{\xi < t\}$). In the case of $F(x,t) \neq L(x)B(t)$ the solution is possible for queues similar to $M/G/n/0$, $M/G/\infty$ classical models and for queues similar to classical processor sharing models. Models of this class sufficiently describe service processes in communication nodes.

2. Models of queues with unrestricted memory volume ($V = \infty$) and dependent random variables ζ and ξ ($F(x,t) \neq L(x)B(t)$). For such models in many cases characteristics of $\sigma(t)$ process can be obtained.

Note, that in the case of $F(x,t) = L(x)B(t)$ and $V = \infty$ the problem becomes trivial.

To demonstrate the importance of solving the discussed problem let us consider two different $M/M/1/\infty$ queueing systems with unrestricted summarized volume.

In both systems demands volume has an exponential distribution with parameter $f > 0$ ($L(x) = 1 - e^{-fx}$). In the first system service time is independent of demand volume and has an exponential distribution with a parameter $\mu > 0$ ($B_1(t) = 1 - e^{-\mu t}$). Then for the first system we have $F_1(x,t) = L(x)B_1(t) = (1 - e^{-fx})(1 - e^{-\mu t})$. In the second system service time is proportional to the demand volume, i.e. $\xi = c\zeta, c > 0$, so that

$$B_2(t) = \mathbf{P}\{\xi < t\} = \mathbf{P}\{c\zeta < t\} = \mathbf{P}\left\{\zeta < \frac{t}{c}\right\} = L\left(\frac{t}{c}\right) = 1 = e^{-ft/c}.$$

Let $\mu = f/c$. Then the both systems become equivalent from the point of view of classical queueing theory.

But for the first stationary moment of summarized volume $\mathbf{E}\sigma$ ($\sigma(t) \Rightarrow \sigma$ in the sense of a weak convergence) we have respectively:

$$\mathbf{E}\sigma_1 = \frac{1}{f} \cdot \frac{\rho}{1-\rho}, \mathbf{E}\sigma_2 = \frac{1}{f} \cdot \frac{\rho(2-\rho)}{1-\rho},$$

where $\rho = a/\mu = af/c < 1$, a is a parameter of an entrance flow.

So we can see, that the mean stationary volume in the second queue is $2 - \rho$ time more that one in the first queue.

This simple example demonstrates us, that it is important to take into consideration the dependences between demand volume and it's service time when designing information and communicating systems.

References

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