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The Phenomena of the Infi-calculus in Elementary Mathematics

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Introduction

This article introduces the results of a research taken in 1998 and 1999 on primary schools in the region of North Bohemia. The aim of this research was to describe the notions the pupils of primary schools and students of secondary schools have about these phenomena: *power of a set, numerical density, maximum and minimum of a set, convergence and straight line and their development* of these notions through the school attendance. The test was given in April 1999 on 12 different schools in northern Bohemia to approximately 300 respondents - of each the 1^{st}, 3^{rd}, 5^{th}, 7^{th} and 9^{th} year of primary schools and the 2^{nd} and 4^{th} year of upper secondary schools. In this article I only deal with the evaluation of the results of pupils from the 1^{st}, 3^{rd} and 5^{th} year of primary schools. In the 1^{st} year the test had the form of an interview with each pupil alone, the answers were recorded on a Dictaphone. In the 3^{rd} and 5^{th} year the test was in a written form, pupils filled in a pre-set form anonymously.

Results

The summarised answers of pupils of the 1^{st} and 3^{rd} year are shown in the charts. Isolated uninteresting answers are not mentioned. The symbol o means that e.g. question no. 12 was answered by 13 % of pupils of the 3^{rd} year „4", „5" or „6". The answers „A lot" and „Many" were not distinguished in the summary, they are put under the heading „A lot". Nor are the answers „Infinity" and „∞" put separately in the charts.

The 5^{th} year pupils' results are not explicitly shown according to the restricted extension of this article, they are, however, included in the column graphs showing the development of the primary school pupils' notions of

some of the phenomena mentioned above. From the results, the development of which can be traced to the upper secondary school, we assume, for example, that during their education the pupils tend more and more to consider the infinity as an element of the set of natural, rational or real numbers or at least treat it like that. This conclusion is in agreement with the results found in the 80-ties and 90-ties in Germany [1, 2].

References

[1] Herden, G.; Knoche, N.; Pickart, U.: Eine Untersuchung zur Diskussion über Schwierigkeiten im Umgang mit dem Konvergenzbegriff, JMD 4 (1983), 263 - 305

[2] Zeitz, G.: Zur Entwicklung und Überprüfung des „infiniten Denkens", Mathematik in der Schule 28 (1990), 778 - 783

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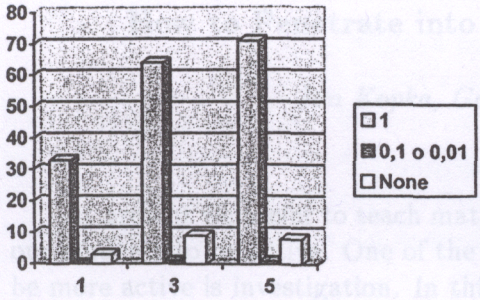
1st year

1. To how many can you count?	20	10	100	11 o 12 o 13 o 15	1000	I do not know			
%	30	17	17	10	9	7			
2. What highest number do you know?	20	10	I do not know	11 o 12 o 13 o 16	100	Million	1000	Infinity	50
%	14	14	12	10	10	9	8	5	5
3. If we count 1, 2, 3, 4, etc. on and on when will we end?	I do not know	20	10	100	In infinity	Never	Milion	At the end	
%	39	16	9	7	4	3	3	1	
4. What is the smallest number bigger than 0?	1	I do not know	0	10	3	None			
%	33	23	8	5	5	3			
5. What is your favourite toy?	Dolls	Car	Teddy Bear	Robot	PC				
%	25	17	14	7	4				
6. Is there infinitely many people on the Earth?	I do not know	Yes	No						
%	44	31	18						
7. Do you know anything that is infinitely big?	I do not know	Earth	Space	Giant	Block of flats				
%	52	16	10	5	5				
8. Do you know anything that will never end?	I do not know	Earth	Space	Television	World	Counting	Life	Numbers	TV series
%	62	8	5	3	3	1	1	1	1

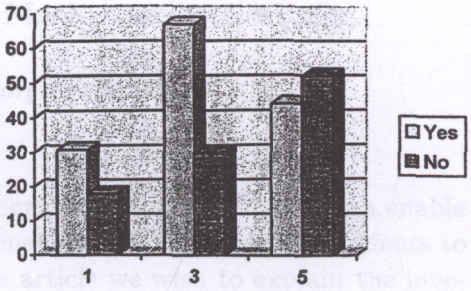
3rd year

1. To how many can you count?	1000	100	Million	10 000	To how many I wish			
%	54	14	11	8	1			
2. What highest number do you know?	Million	Infinity	1 000	Trillion	100 000	Milliard		
%	37	14	11	8	5	4		
3. If we count 1, 2, 3, 4, etc. on and on when will we end?	Never	I do not know	In infinity	100	1000	10	Million	Any time
%	30	22	12	11	9	5	3	2
4. What is the smallest number bigger than 0?	1	None	I do not know	0,1				
%	64	9	8	1				
5. Of what there is infinitely many?	Numbers	People	I do not know	Trees	Water	Stars		
%	24	23	23	5	3	2		
6. Is there infinitely many people on the Earth?	Yes	No	I do not know					
%	67	30	2					
7. Is there infinitely many sand grains on the Earth?	Yes	No	I do not know					
%	70	25	5					
8. Do you know anything that is infinitely big?	I do not know	Space	Earth	Skyscraper	World	Sun	Sea	Straight line
%	29	20	19	6	3	3	2	2
9. Do you know anything that will never end?	I do not know	Space	World	Sky	Straight line	Time	Numbers	Life
%	37	15	8	6	5	5	5	4
10. What is a straight line?	Straight line	Line	Endless line	Long line	I do not know	Endless abscissa		
%	31	21	16	12	9	2		
11. How many straight lines can cross one point?	3 o 4 o 5 o 6	1	I do not know	A lot	Infinitely many	2	Arbitrarily many	
%	30	13	12	11	8	8	6	
12. How many straight lines can be parallel to one straight line?	I do not know	4 o 5 o 6	Infinitely many	1	A lot			
%	45	13	11	8	8			

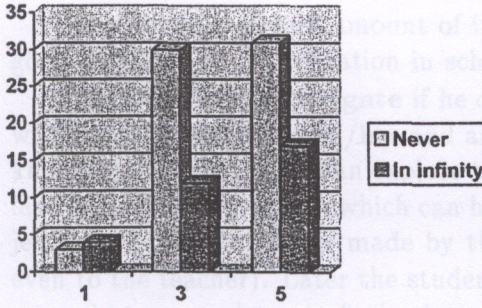
What is the smallest number bigger than 0?



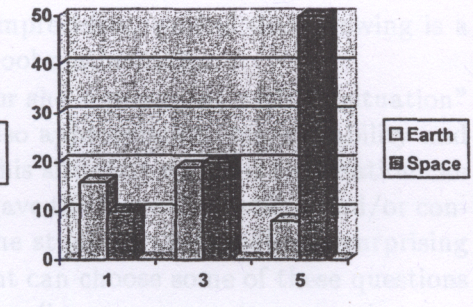
Is there infinitely many people on the Earth?



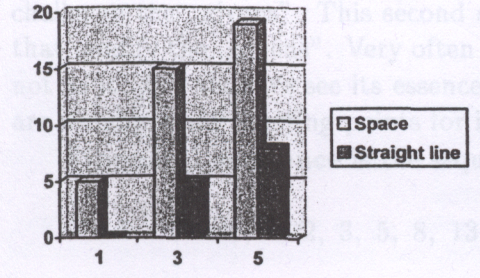
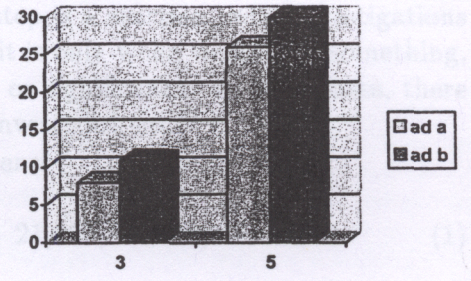
If we count 1, 2, 3, 4, etc. on and on when will we end?



Do you know anything that is infinitely big?



Do you know anything that will never end?

a) How many straight lines can cross one point?
b) How many straight lines can be parallel to one straight line?

Answer: Infinitely many