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Using Modern Technology to Teach Precalculus

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Calculators, particularly graphics calculators or even computer algebra systems calculators (CAS) present a dramatic new challenge in teaching mathematics. These new tools provide a large set of opportunities for teachers for relation to what is taught, and why and how it is taught; they change activities in the mathematics classroom and suggest issues that must be considered in designing curriculus and assessment strategies. Just as the four-function calculators challenged the role of pencil-paper skills in arithmetic and the goals of elementary school mathematics – these a basic level calculators increase and speed up calculating power and widen the range of calculations which students can manage, graphics and CAS calculators are forcing a serious examination of secondary schools and to a lesser extent of colleges and universities.

The main purpose of this paper is to introduce mathematical ideas and techniques that have been beyond the reach of the concept of a function.

The early history of the development of that one from Newton to Cauchy was dominated by two ideas – that of algebraic construction and that kinematic continuity. Starting from unanalysed idea of a „real variable" capable of taking all real values in some interval, the algebraic processes of arithmetic operations together the extraction of square roots and the solution of algebraic equations yielded familiar and known constructions for a large variety of functions. On the other hand, the concept of the motion of a particle along a straight line introduced an idea of geometric or kinematic continuity which persisted until Darboux proved that it was inadequate for the purposes of analysis.

A third tributary to the main stream of function theory was the subject of the trigonometric functions which possessed such easy geometric definitions, enjoyed very many interesting algebraic properties.

All three tributaries converged in the investigations of the problems of vibrating cords which compelled the consideration of „arbitrary" functions which were neither algebraic, nor kinematic, nor trigonometric, but which could be represented by infinite, convergent series of trigonometric functions.

The formal definition of a function of a real variable as a subset of the cartesian product space which is ,,single-valued" tells us without doubt everything we need to know about the function as a basis for a rigorous theory, but generations of students have found it unacceptaly usefulness after the vague but evocative descriptions given by Newton, d'Alembert and Euler.

Let us go back therefore to the function concept taught is currently being taught at most secondary and post-secondary institutions. It is again well known this concept has not always been well understood and this misunderstanding frequently results in low achievement in precalculus. This, in turn, creates obvious difficulties in subsequent college mathematics courses, such as calculus, which require a sound understanding of elementary functions and their graphs.

Pencil-and-paper graphing methods are often employed to graph functions by plotting a small sample of points whose co-ordinate satisfy the defining equation of the function. Such methods are both time-consuming and frequently inaccurate. Futhermore, when such inexact methods are used, the graph often becomes the main focal point and the global characteristics of the function being studied gets lost in finding and plotting points. It has been asserted that these typical pencil-and-paper methods are sometimes inadequate to solve many of the more interesting and practical problems which can be modeled with elementary functions.

Therefore some mathematicians and mathematics educators are turning their attention during last years to the potential use of modern technology in the teaching and learning of functions and their graphs (and other related precalculus concepts). More recently, the price of graphing calculators has dropped to the point where students can afford to purchase their own.

The most obvious benefit of graphing calculators is the ease with which they produce complete pictures for a variety of functions. Graphics and CAS calculators then allow students to graph a function quickly, to isolate a section of the graph, to zoom in for greater detail, to zoom out to view the function as x increases or decreases, and to compare the graph of one function with the graph of another function. With little training, students using these calculators can graph families of curves and discover the connections between algebraic and graphical representations of functions.

Computer algebra systems machines provide a learning environment in mathematics which is interactive and dynamic; it is possible to produce animated Graphic Interchange Format (GIF) images. These consist of a set of individual GIFs which are overlaid in quick succession like ,,frames" in a film. If the individual images are essentially identical apart from one element which changes position from frame to frame, then the animated GIF will appear to show that element moving in real time. The actual animation is produced from the individual images using a special piece of software. GIF animation programs all follow the same basic structure. They require the individual GIF images to have been created first and then copied into the animation program as „frames" for the final animation.

In what follows, we give an example in which the animation can be used on the Texas TI-92 CAS calculators to provide a dynamic image which can illustrate mathematics in a qualitatively superior way to the static images used in traditional paper-based teaching materials.

Modeling Projectile Motion

When we shoot a projectile into the air we usually want to know beforehand how far it will go (will it reach the target?), how high it will rise (will it clear the hill?), and when it will land (when do we get results?). We get this information from equations that calculate the answers we want from direction and magnitude of the projectile's initial velocity vector, described in terms of the angle and speed at which the projectile is fired. The equations come from combining calculus and Newton's second law of motion in vector form. To derive these equations for projectile motion, we assume that the projectile behaves like a particle moving in a vertical coordinate plane and that only force acting on the projectile during its flight is the constant force of gravity, which always points straight down. (In practise, none of these assumptions really holds. The ground moves beneath the projectile as the earths turns, the air creates a frictional force that varies with the projectile's speed and altitude, and the force of gravity changes as the projectile moves along.)

As the result we will find the parametric equations

$$x = (v_0 \cos \alpha)t \qquad y = (v_0 \sin \alpha)t - \frac{1}{2}gt^2$$

express the coordinates of the projectile's position in terms of the initial speed v_0 and angle of elevation or firing angle α (g is the acceleration of gravity).

We can see from the parametric equations (if we substitute t from the first equation into the second) that ideal projectiles move along parabolas and it is easy to make a hand-drawn graph for this projectile.

Consider now the situation where two objects are projected towards each other with different initial velocities. The problem is to establish whether they collide.

A common mistake made by students in this type of problem is to find the equations of the two parabolic flight paths in the form $y = f(x)$ and calculate their point of intersection as the point of collision. This error would be reinforced by the kind of static diagram which is often shown in textbooks.

On the other hand, if we use the parametric equations of motion and the animation on the Texas TI-92 calculator, we can see what happens ,,in fact".

In the conclusion we'd like to repeat that in our opinion graphics calculators are at the present time available and inexpensive enough to allow students to have access to them for learning about elementary functions. Their use enables students to construct many more graphs for observation and generalization than they would usually do by hand. Instruction that connects graphical, algebraic and tabular representations of functions helps students develop richer insight into the nature of functions. In particular, by removing much of the drudgery of routine calculations, they allow more realistic problems to be tackled, with greater concentration on the modeling process at more advanced levels.

References

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