

Noise Reduction in High Resolution ECG by Using Frequency-domain Adaptive Filtering

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Abstract — In this paper a frequency-domain least-mean-square adaptive filter (FDLMS) is used to cancel noise in real time recording of the high resolution ECG (HRECG). In this paper, beside the classical FDLMS solutions, an application of the network of the time sequenced adaptive recurrent filter is shown. Such a filter requires of an additional synchronisation algorithm, however, is able to tract rapidly varying nonstationarities without smoothing effect and therefore can be used for dynamically changing ECG signals. The results of our experiment show the importance of using digital signal processing when dealing with HRECG signal corrupted by noise.

1. Introduction

The adaptive signal processing technique appears to be appropriate for time-varying situations. The adaptive filtering can be applied to both kinds of the signal: to the signal of continuing character (like EEG, ENG) and to quasi periodic signal (like ECG) and also to dynamically changing quasi periodic signal (like exercise ECG). Adaptive filters are self-designing ones based on an algorithm which allows the filter to learn the initial input statistics and to track them if they are time-varying [4]. These filters estimate the deterministic signal and remove the noise uncorrelated with the deterministic signal. HRECG is associated with high amplification of the ECG signal and is always corrupted by different kinds of noise — such as EMG (muscle noise), not easy to remove, because of overlapping of the electrogram bandwidth. The relevant signal is buried in a noise background where we have little or no prior knowledge of the noise characteristics. This is why it is very difficult to eliminate the interference by using classical approach to filtering, and the background noise causes serious difficulties. Therefore, in order to get the best estimation of the corrupted signal, an adaptive noise canceler is used.

2. Comparison of the adaptive filter with an ensemble averaging

The adaptive filter has two or more inputs. The primary signal which contains noise is applied to the primary input. Noise components are fed to reference input (or inputs). The reference signal must be a correlated version of the noise that is present in the primary signal. Another adaptive approach (which has been applied for clinical applications) consider that the signal is recurrent and the noise is random and Gaussian. Thus, both inputs of the filter (the primary and the reference) are the same, but the former is a delayed version of the latter. In this paper we analyse a time-sequenced adaptive recurrent filter (ARF). The ARF can be applied to evoked potentials and low-amplitude potentials that are time-locked to the high-amplitude wave of the HRECG (late potentials and His-Purkinje potentials).

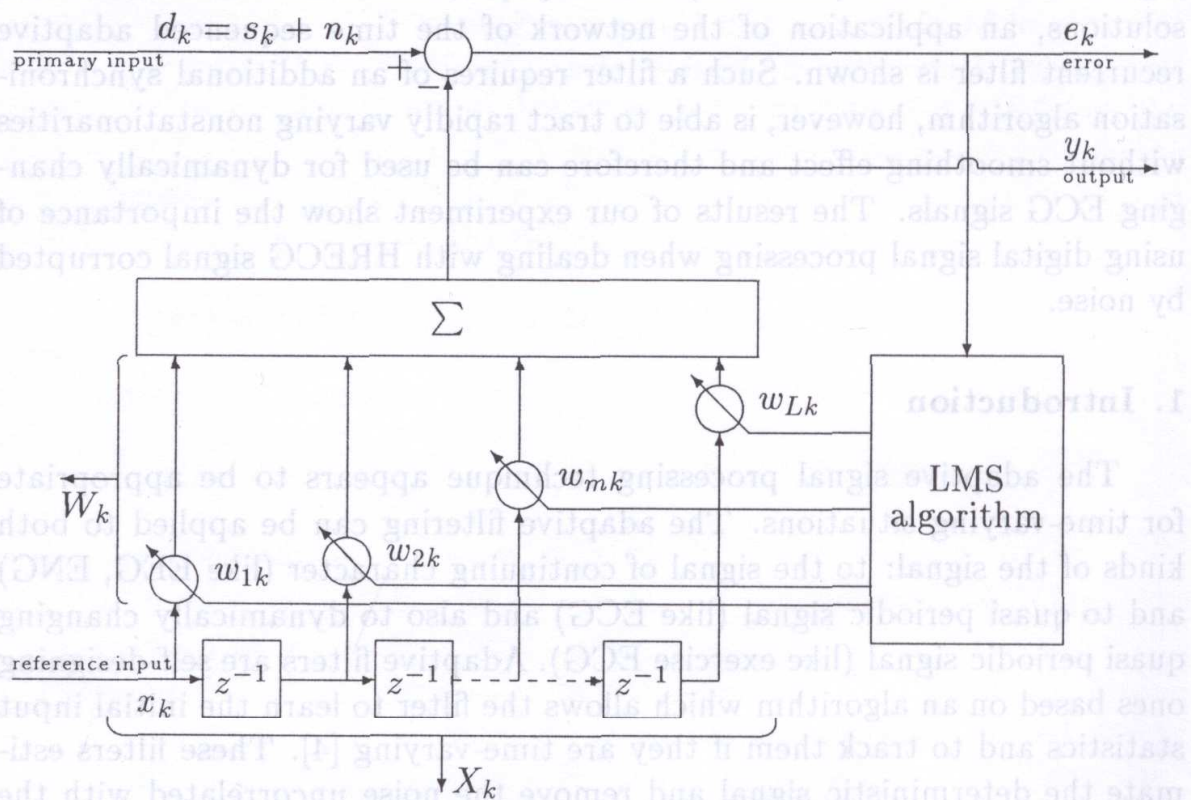


Fig. 1. Block diagram of ARF

The ARF needs two inputs: the signal $d_k = s_k + n_k$ (primary input which contains noise) and the reference input x_k correlated with the deterministic component (Fig. 1). The reference input of the ARF x_k is a unit impulse sequence synchronized with the beginning of each recurrence of signal. The output of ARF y_k can be expressed, according to classical FDLMS notation, by

$$y_k = W_k^T X_k,$$

where W_k is the weight vector and X_k is the reference vector. The implementation of ARF and its mathematical expressions become especially simple when the reference input is an impulse and the FDLMS algorithm is used in the adaptation process [3].

The HRECG is associated with high amplification of the ECG signal and is always corrupted by different kinds of noise. Noise and artefacts cannot be effectively suppressed by the bandwidth limiting analogue and/or digital filters. In the majority of applications associated with measurement of the HRECG signal, the digital averaging is a standard method for improvement of the signal to noise ratio (SNR). The classical ensemble averaging (EA) technique is a method for recovering the signal hidden in the noise, but it needs a large number of records to obtain good estimation of the signal. We show that ARF (when the LMS algorithm is used in the adaptation process) is equivalent to classical EA (for integral number of occurrences N). In order to determine the performance of ARF, the SNR improvement in the case of a stationary deterministic signal s_k will be studied. The SNR_d at primary input signal d_k can be defined as follows:

$$SNR_d = \frac{E_t[s_k^2]}{E_t[n_k^2]},$$

where expectation $E_t[\]$ represents the mean of all the possible values as a function of time for one recurrence of the process; this mean does not depend on the chosen recurrence if the ergodicity can be assumed [2]. When the LMS algorithm has not yet converged, the output signal y_k can be considered to be composed of a signal component s'_k (correlated with s_k) and a noise component n'_k (uncorrelated with s_k). That is

$$y_k = s'_k + n'_k.$$

To evaluate SNR_y as a function of SNR_d it is necessary to determine relation of $E_t[s_k'^2]$ and $E_t[n_k'^2]$ with $E_t[s_k^2]$ and $E_t[n_k^2]$, respectively. The relation between $E_t[n_k'^2]$ and $E_t[n_k^2]$ is given by

$$E_t[n_k'^2] = \mu E_t[n_k^2] (1 - (1 - 2\frac{\mu}{L})^{2k}),$$

where μ is the gain constant of LMS algorithm. Now the SNR_y can be described by:

$$SNR_y = \frac{E_t[s_k'^2]}{E_t[n_k'^2]} = \frac{SNR_d (1 - (1 - 2\frac{\mu}{L})^k)^2}{\mu (1 - (1 - 2\frac{\mu}{L})^{2k})}.$$

The improvement of SNR can be presented by the expression:

$$\Delta SNR_y = \frac{SNR_y}{SNR_d} = \frac{1 (1 - (1 - 2\frac{\mu}{L})^k)^2}{\mu (1 - (1 - 2\frac{\mu}{L})^{2k})}.$$

This expression can be reformulated with N (the number of occurrences):

$$\Delta SNR_y = \frac{1 (1 - (1 - 2\frac{\mu}{L})^{NL})^2}{\mu (1 - (1 - 2\frac{\mu}{L})^{2NL})}$$

The convergence condition of LMS assures that:

$$|1 - 2\mu/L| < 1.$$

When $N \rightarrow \infty$ the steady-state improvement of ΔSNR_y is reached:

$$\lim_{N \rightarrow \infty} \Delta SNR_y = \frac{1}{\mu}.$$

In classical EA the improvement of the SNR increases indefinitely with N , while in the ARF the SNR reaches a constant value $\frac{1}{\mu}$. The advantage of ARF is the capability of adaptation to dynamic changes in the deterministic signal s_k . EA does not adapt to such changes. The ARF algorithm requires the knowledge the signal episode beginning location with respect to time, which allows synchronization of the filtering [1].

3. Conclusion

The aim of our investigation was to analyse the HRECG signal acquired during exercise in order to complement the standard parameters of a static character by the results associated with the load application. We have shown the following facts:

1. Adaptive algorithm based on the FDLMS seems to be a promising method for the signal to noise improvement in the HRECG signals (under condition that the filter parameters are carefully chosen).
2. ARF is superior to the classical adaptive FDLMS algorithm if dynamically changing quasi-periodic signal is to be recorded, however, for that technique the additional synchronization algorithm must be applied.
3. Exercise HRECG processed by the ARF filtering system may offer potential for new diagnostic approach, complementary to that of resting signal averaging ECG.

Our simulation results agree with this theoretical study. ARF is able to process the on-going exercise HRECG, which is dynamically modified and corrupted by enhanced level of noise, completely without or very short averaging procedure. Moreover, the ARF algorithm used, is able to extract well pronounced exercise component.

References

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$$y(x, t) \Big|_{t=0} = f(x) \quad -1 < x < 1 \quad (2)$$

is considered.

Usually the diffusion equation (1) is supplemented by the Dirichlet

$$y(x, t) \Big|_{x=-1} = \phi_1(t), \quad 0 < t \leq T, \quad (3)$$

$$y(x, t) \Big|_{x=+1} = \phi_2(t), \quad 0 < t \leq T \quad (4)$$

or Neumanns

$$\frac{dy(x, t)}{dx} \Big|_{x=-1} = \phi_3(t), \quad 0 < t \leq T, \quad (5)$$

$$\frac{dy(x, t)}{dx} \Big|_{x=+1} = \phi_4(t), \quad 0 < t \leq T \quad (6)$$

boundary conditions.

In equation (1) and conditions (2)–(6) $u(x, t)$, $f(x)$, $\phi_1(t)$, $\phi_2(t)$, $\phi_3(t)$, $\phi_4(t)$ are known functions.

Instead of the boundary conditions at $x = -1$ and $x = +1$ ((3)–(4) or (5)–(6)), in this paper the diffusion equation (1) is subject to two integral conditions (specifications on mass in a case of diffusion or specifications on energy in a case of heat transfer)

$$\int_{-1}^{+1} y(x, t) dx = g_1(t), \quad 0 < t \leq T, \quad (7)$$