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## Exponential and Logarithmic Functions

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### Abstract

At the Technical University of Liberec, a new international Bachelor branch of studies was established for students of three states - Czech Republic, Poland, and Germany. These studies are intent on a management. We want to analyze students' understanding of one pivotal mathematical concept that plays a central role in advanced mathematics. We also want to compare a level of knowledge of these students from three different states. We aim at exponential and logarithmic functions because these concepts give students serious difficulties. In this report, we describe how students acquire an understanding of these functions by prescribing a set of mental constructions that students can make to develop their understanding of these concepts.

Exponential and logarithmic functions are pivotal mathematical concepts that play one of important roles in advanced mathematics but these functions and relations between them give students considerable difficulties. The students understanding of exponential and logarithmic functions depends on the understanding of a set of mental constructions touching the logarithms and exponents.

At the Technical University of Liberec, a new international Bachelor branch of studies was established for students of three states - Czech Republic, Poland, and Germany. This six-semester study with the official name „Management of Information and Communication” is shortly said The Nisa University because students of the Nisa region take part in this programme. There are students of the Technical University of Liberec, the universities of Wrocław and Zittaw there. They are in Liberec in the first year, then in Wrocław, and they will graduate in Zittaw. Students of the first year study must pass through the mathematical courses in Liberec, and so we have got a occasion to compare a level of their knowledge in the secondary mathematics.

In the beginning of the first semester, the students filled in our questionnaire in a mathematical lesson. First, we wanted to obtain characteristics of students (types of secondary schools, level of their mathematical knowledge and knowledge of English etc.), and to obtain information whether students read (mathematical) books. We were also interested in students' evaluations of their own knowledge.

After completing questionnaires the students solved seven elementary examples concerning the exponents and logarithms. A few students finished this test before the time limit. Then they were divided into three groups - Czech, German, and Polish. The following part of our investigation was running with each group separately. Students answered the same four questions of the theme. These dialogues were recorded to the dictaphones. Each dialogue could not be longer than six minutes. In the interviews, students were asked a wide range of questions. They were asked to recall properties of exponents and logarithms, explain why these properties were true, and to perform standard and non-standard computation. If student did not answer immediately the following question had been done. The questions and appropriate subsidiary questions were prepared so that we might guarantee the objectivity as it is possible.

Only sixteen students took part in our research only. From questionnaires it follows that one half of them graduated the secondary schools in the last year. It is very interesting that while Czech students were being graduated schools with a view to an economics, the almost all Polish and German students are from grammar schools (gymnasia). Fifteen students enrolled in a pre-calculus course at our university volunteered to participate in this study. Evaluation of their knowledge of English was very positive but students evaluated their mathematical knowledge mediocrity and worse. All students asserted that they read very much, even also mathematical books. It follows from the test that while solving simple exponential equations and examples with properties of powers correctly, they were not able to solve the basic examples with logarithms. With respect to dialogues with students, it is necessary to note, the open-ended questions, that probe students' general understanding of this topic, were difficult for them. Many students did not answer or they answered inexactly.

**Let us see the following example (see Appendix 2):**

T: *What does the function  $f(x) = a^x$  mean to you? What do you think of when you see this function?*

S: I think the name of this function ... I think, it is an exponential function and if this function is increasing or decreasing, it depends on the „ $a$ ”. I know how this function looks in the Cartesian product. I can sketch this.

T: Is  $5^{6789}$  an even or an odd number?

S: I think it is an odd number.

T: Why?

S: Because the end of the exponent is nine and five (he shows five) is odd number too.

T: Is  $f(x) = \left(\frac{1}{2}\right)^x$  an increasing function or decreasing function?

With respect to the third question, almost all students answered correctly that the given function was decreasing. Explaining why this was a decreasing function requires a process understanding of exponentiation. Only 4 students were able to give a mathematical explanation for it.

The last question was also difficult for students. Mostly they knew that must find an  $x$  such that  $5^x$  equals 78 125, but were unable to find a way to determine what this  $x$  was.

In the end, we can summarize that the students cannot formulate properties of exponential and logarithmic functions and they have the fragmentary knowledge only. They cannot give reasons for their correct answers. No differences between Czech and foreign students be revealed. We take notice of a fact that the number of investigated students were not so big, but nevertheless we think that our investigation can present something interesting. This will continue in investigation of students' imagination of the function.

## References

- [1] D. Bittnerová, *Univerzita Nisa versus exponenciální a logaritmické funkce*. In: XXI vědecké kolokvium o řízení osvojovacího procesu. Vyškov 2003.
- [2] K. Weber, *Students' Understanding of Exponential and Logarithmic Functions*. ICTM 2, Crete 2002.

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## Appendix 1

## Exponential and Logarithmic Functions

Denote your choice by a cross: 

QUESTIONNAIRE					U Nisa	Date	Time				
Code	(Fe)Male	F	M	First name	Surname						
Country			Type of secondary school								
CZ	G	PL	Gymn.	Vocational	Sch. of Econ.		Another				
The year of your <i>leaving examination</i> (the graduation) at the secondary school					1998	1999	2000	2001	2002		
Was Maths a <i>part</i> of your <i>leaving examination</i> ?					yes		no				
Did you take in another <i>preparing</i> mathematical <i>course</i> (different from the TUL course)?					yes		no				
When did you have the <i>last math-lesson</i> (before your entrance examination).					1998	1999	2000	2001	2002		
Estimate the level of your <i>knowledge</i> of the secondary <i>mathematics</i> .					oral		--	-	0	+	++
					written		--	-	0	+	++
Estimate the level of your knowledge of <i>English</i> .					oral		--	-	0	+	++
					written		--	-	0	+	++
Do you <i>like Mathematics</i> ?							--	-	0	+	++
How many <i>pages</i> have you <i>read</i> ?					text		math.		another		
					language						
					English						
					native						

## Appendix 2

## Solve the following examples:

1) Solve these equations with respect to  $x$  :

a)  $3^x = 81$

b)  $5^x = 1$

2) Evaluate:

a)  $\left(\frac{49}{16}\right)^{-\frac{1}{2}}$

b)  $(64)^{-\frac{1}{3}}$

c)  $\log_3 9$

3) Express in terms of  $\log p$ ,  $\log q$ , and  $\log r$  :

a)  $\log(p^2q)$

b)  $\log \sqrt{\frac{q}{r}}$

## Interview:

1) What does the function  $f(x) = a^x$  mean to you?

What do you think of when you see this function?

2) Is  $5^{6789}$  an even or an odd number?

3) Is  $f(x) = \left(\frac{1}{2}\right)^x$  an increasing function or a decreasing function?

4) Suppose you didn't have a calculator. How would you go about computing  $\log_5 78125$ ?

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