

Geometrical Folder

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Abstract. The contribution deals with demonstration of “Geometrical folder” in plane and space as a tool for pupil’s imagination development. The example of task in the plane is to fold step by step various plane patterns from a set of “universal” patterns as parts. Analogically, the example of task in space is to complete specified body from a set of various parts. Using this exercises I investigate a difference of a space imagination between boys and girls, and between younger and older students.

Space imagination of kids plays an important role in teaching mathematics and especially geometry. According to recent research the level of space imagination of pupils decreases, which brings forward the necessity to develop it on every occasion. Geometrical puzzles, where pupils are requested to assemble patterns or bodies from compounds, are one of the options. Space imagination activities are complemented with manual activities and experiments leading to increased pupil’s activity during lessons.

Geometrical imagination can be developed by e.g. puzzle in a plane, where a set of 5 “universal” patterns (Fig. 1) is given to pupils. Pupils are requested to subsequently assemble (a) square, (b) rectangle, (c) triangle, (d) parallelepiped, (e) trapezium, (f) quadrilateral. All patterns of the set have to be used in all cases.

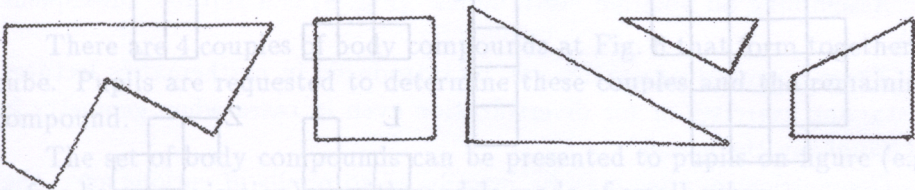


Fig. 1.

Set of patterns may be paper or plastic made. For younger pupils the tasks (d), (e), and (f) may be assigned by words “find other patterns that can be assembled from the set” or the shape of the patterns may be defined by figures.

Solution

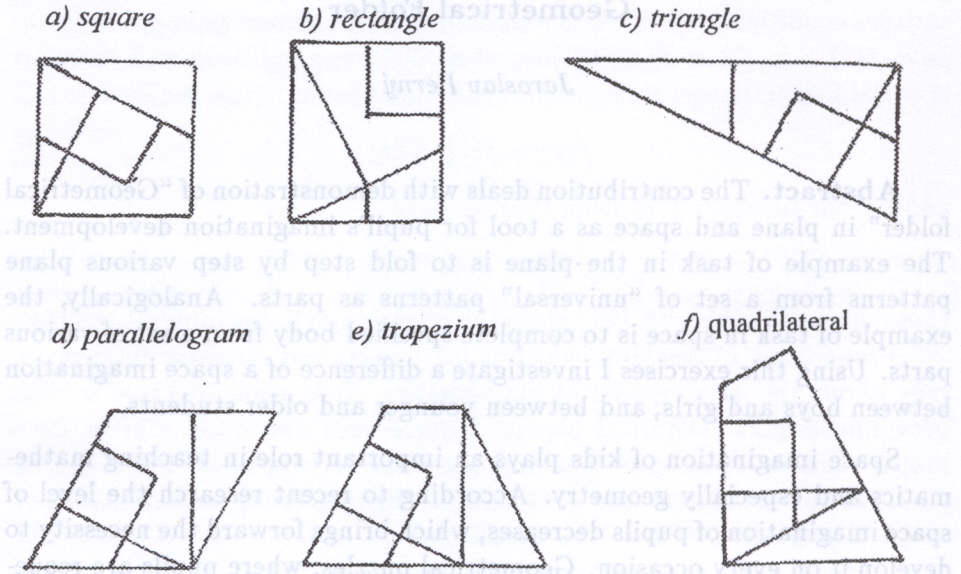


Fig. 2.

There are also other possible plane mathematical puzzles. In one of these tasks pupils are requested to assemble defined pattern (Fig. 3) from plane “tetramines”. Plane “tetramino” is a pattern composed of 4 equal squares with common side. Plane “tetramines” may be made of paper or plastic. Assembling all possible “tetramines” (Fig. 4) may be a partial task.

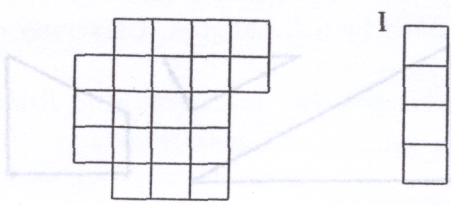


Fig. 3

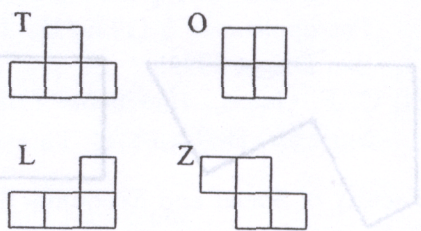


Fig. 4.

Solution

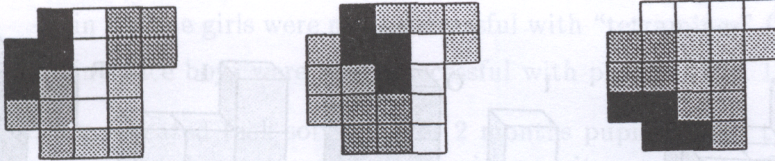


Fig. 5.

Space imagination can be developed by space puzzles. Pupils are requested to assemble complete body from provided set of body compounds (Fig. 6), nothing may be missing or left over.

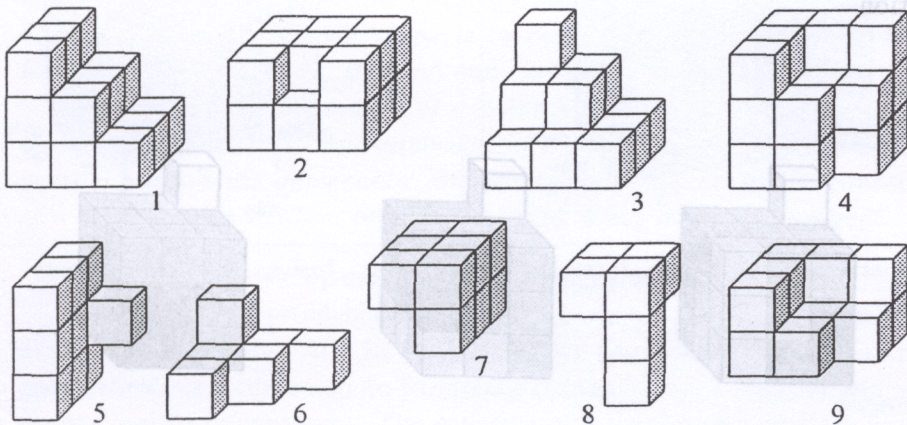


Fig. 6

There are 4 couples of body compounds at Fig. 6 that form together a cube. Pupils are requested to determine these couples and the remaining compound.

The set of body compounds can be presented to pupils on figure (e.g. in free linear projection) or with models made of small cubes.

Like in a plane, there is an analogical task with space "tetramines" where pupils are requested to assemble defined body (Fig. 7). Are there

more ways to assemble this body? Space “tetramines” are bodies compound of 4 equal cubes with common side. “Tetramines” should be modeled from cubes for this task.

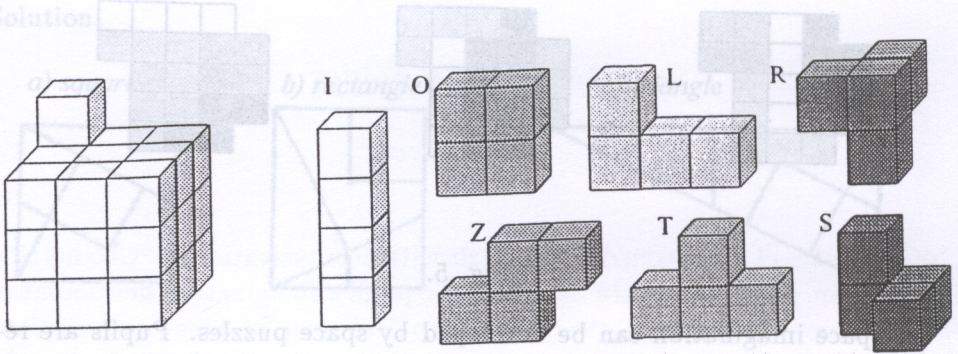


Fig. 7.

Fig. 8.

Solution

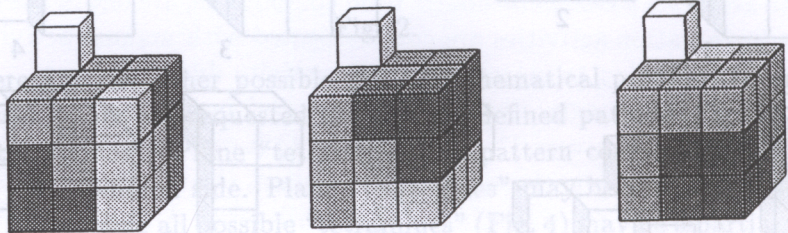


Fig. 9.

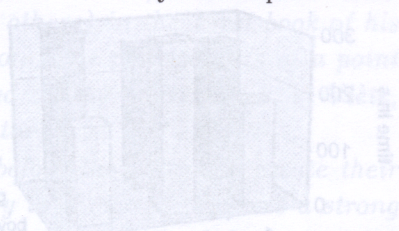
Assembling all possible “tetramines” (Fig. 8) is a suitable subsequent task.

These geometrical puzzles can be introduced in mathematics lectures as starting exercises or for diversification even in cases when geometry is not taught in the lesson.

Geometrical puzzles may be introduced to pupils of various ages; the setting has to be adapted and suitable tools have to be selected according to pupil's age.

Some results: (experiments were carried out with students of the 5th class)

- task solving
 - in a plane girls were more successful with “tetramines” (Fig. 10),
 - in space boys were more successful with puzzles (Fig. 12);
- during repeated task solving after 2 months pupils stated that they do not remember any previous experiments; it was obvious for some, but the general successfulness of solving increased despite pupils did not notice it (“tacit knowledge”)
 - in a plane boys were more successful with puzzles (Fig. 11),
 - in space girls were more successful with “tetramines” (Fig. 13);
- from Figs. 10 - 13 it is obvious that increase in successfulness in repeated experiment was more significant for boys in a plane and girls in space.



Legend: Task 1 Puzzle, Task 2 Tetramines

Constructional problems were the main concern of Greek geometry in 6th and 5th centuries BC. In this period we can find attempts to create geometrical structures and to formulate opinions about what is and what is not possible to construct. The concepts of a point and a straight line are not ideal objects.

Pythagoras' school understands space as a sum of points which has a position. A point is thus a basic unit which has a placement. [1] J. Jankovský, *Geometrie*, Smeř (1985), p. 42].

Parmenides (from Elea, about 530 - 470 BC) cogitates on how the world is recognizable. He compares the sensory and intellectual cognition. For example: What is a point? A dot on a paper? Or a real geometrical object, it is only its model, its picture [Hejný 1985, pp. 50-51].

Fig. 10

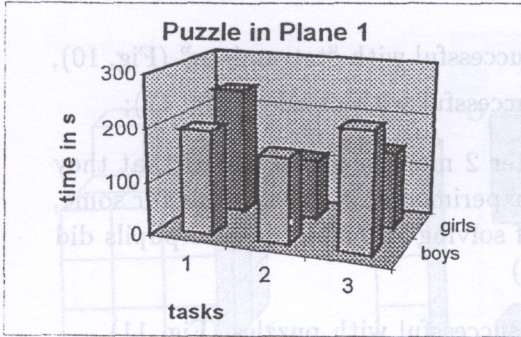


Fig. 11

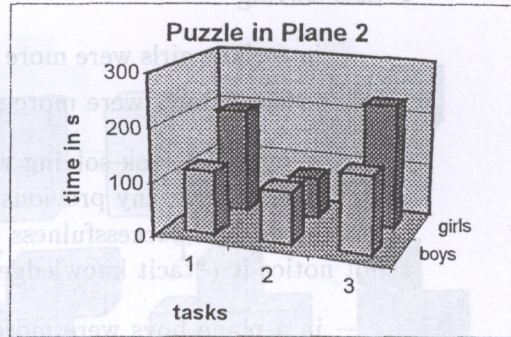


Fig. 12

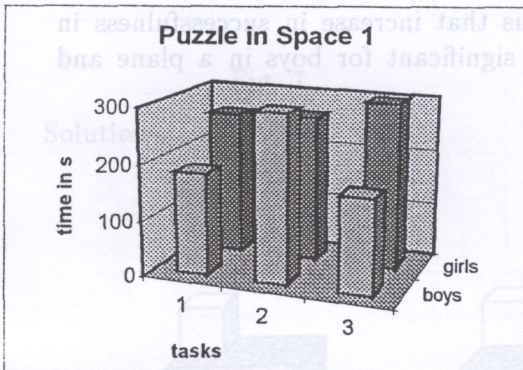
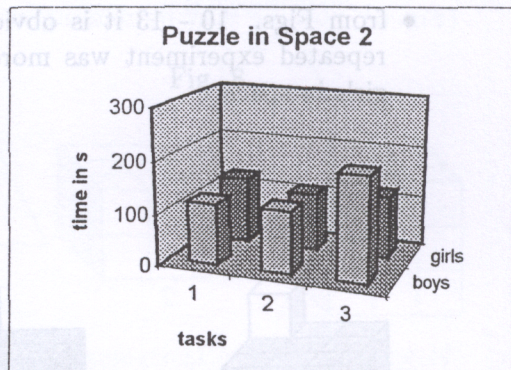


Fig. 13



Legend: Task 1 Puzzle, Task 2 „Tetramino“ - assembling, Task 3 „Tetramino“ - puzzle

References

- [1] D. Vondráčková, *Vhled žáků do prostorové představivosti*. Diploma thesis, Liberec 1998.
- [2] Z. Jarkovská, *Geometrická skládanka*. Semestral thesis, Liberec 2002.

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