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Remarks on permutable n-groups

Wiesław A. Dudek and Zoran Stojaković

Introduction. In [8] σ-permutable n-groupoids, which represent a generalization of several classes of n-groupoids, were considered. Some well known classes of n-groupoids are special cases of σ-permutable n-groupoids. Among them are (i,j)-commutative and commutative n-groupoids, cyclic n-quasigroups from [12], [14], [19] and [20], i-permutable n-groupoids from [7] and [15], medial n-groups (cf. [4], [9]), alternating symmetric n-quasigroups (cf. [3], [13], [18]), parastrophy invariant n-quasigroups (cf. [10], [11] and [17]), totally symmetric n-quasigroups and some others.

Binary groupoids which are σ-permutable for different values of σ are commutative groupoids, semisymmetric groupoids (i.e groupoids satisfying the identity $(xy)x = y$), totally symmetric quasigroups, groupoids satisfying Sade's left key'slaw $x(xy) = y$ and Sade's right key'slaw $(xy)y = x$ (cf. [1]).

This paper is a continuation of [8] and [16]. The primary question which is considered here is the derivability of such n-groups from binary groups. Some question on isomorphisms of σ-permutable n-groups were also considered, but first we give some necessary definitions and notations.

1. Notations and definitions

To avoid repetitions we assume throughout the whole text that $n > 2$. We shall use the standard abreviated notation

$$f(x_1,...,x_k,x_{k+1},...,x_{k+s},x_{k+s+1},...,x_n) = f(x_1^k, \overset{(s)}{x}, x_{k+s+1}^n),$$

whenever $x_{k+1} = x_{k+2} = ... = x_{k+s} = x$ (x_i^j is the empty symbol for $i > j$ and for $i > n$, also $\overset{(0)}{x}$ is the empty symbol).

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If $\sigma \in S_n$, where S_n denotes the symmetric group of degree n, then $x_{\sigma(i)}, x_{\sigma(i+1)}, \dots, x_{\sigma(j)}$ is denoted by $x_{\sigma(i)}^{\sigma(j)}$. If $i > j$ then the symbol $x_{\sigma(i)}^{\sigma(j)}$ is considered empty.

By ω we shall always denote the automorphism $x \mapsto x^{-1}$ of a commutative group. If G is a set, then id denotes the identity mapping of G.

If an n-group (G, f) has the form

$$f(x_1^n) = x_1 \cdot \theta x_2 \cdot \theta^2 x_3 \cdot \ldots \cdot \theta^{n-1} x_n \cdot b\,,$$

where $(G, \cdot)$ is a binary group, $b \in G$, and θ is an automorphism of $(G, \cdot)$ such that $\theta(b) = b$, $\theta^{n-1}(x) = bxb^{-1}$ for all $x \in G$, then this n-group is called *$< \theta, b >$-derived from* $(G, \cdot)$ and is denoted by $der_{\theta,b}(G, \cdot)$. If $\theta = id$ and b is the neutral element of $(G, \cdot)$, then an n-group $(G, f) = der_{\theta,b}(G, \cdot)$ is called *derived from* $(G, \cdot)$ or *reducible to* $(G, \cdot)$. By Hosszú theorem *every binary group* $(G, \cdot)$ *induces on* G *the n-group structure* and conversely, *every n-group* (G, f) *is* $< \theta, b >$*-derived from some binary group defined on* G.

Let (G, f) be an n-group, $a_2, \dots, a_{n-1}$ be fixed elements of G and let $x \cdot y = f(x, a_2^{n-1}, y)$. Then $(G, \cdot)$ is a group which is called *a binary retract of* (G, f). As it is proved in [21] (cf. also [6]) *all retracts of an n-group* $< \theta, b >$*-derived from a given group* $(G, \cdot)$ *are isomorphic to* $(G, \cdot)$. Thus all retracts of a given n-group are isomorphic.

An n-group (G, f) is called *medial* (or *abelian*), if

$$f(\{f(x_{1i}^{ni})\}_{i=1}^{i=n}) = f(\{f(x_{i1}^{in})\}_{i=1}^{i=n})$$

for all $x_{ij} \in G$, $i, j \in N_n = \{1, 2, \dots, n\}$. An n-group (G, f) is medial iff it is $(1, n)$-commutative, i.e. iff $f(x_1^n) = f(x_n, x_2^{n-1}, x_1)$ for all $x_1, x_2, \dots, x_n \in G$ (cf. [9]). In other words, *an n-group is medial iff it has an abelian retract.*

Definition 1. (see [8] or [16]) *Let* $\sigma \in S_{n+1}$. *An n-groupoid* $(G, f))$ *is called σ-permutable iff for all* $x_1, x_2, \dots, x_{n+1} \in G$

$$f(x_1^n) = x_{n+1} \Longleftrightarrow f(x_{\sigma(1)}^{\sigma(n)}) = x_{\sigma(n+1)}\,.$$

An equivalent form of the preceding definition is the following.

Definition 2. *Let* $\sigma \in S_{n+1}$. *If* $\sigma(i) = n + 1$ *for some* $i \in N_n$, *then an n-groupoid* (G, f) *is σ-permutable iff for all* $x_i \in G$, $i \in N_n$

$$f(x_{\sigma(1)}^{\sigma(i-1)}, f(x_1^n), x_{\sigma(i+1)}^{\sigma(n)}) = x_{\sigma(n+1)}\,.$$

If $\sigma(n+1)=n+1$, *then* (G,f) *is* σ*-permutable iff for all* $x_1,x_2,...,x_{n+1}\in G$

$$f(x_{\sigma(1)}^{\sigma(n)}) = f(x_1^n)\,.$$

A σ-permutable n-groupoid which is an n-group is called *a* σ*-permutable* n*-group*.

Examples

(1) Medial n-groups are σ-permutable, where $\sigma=(1,n)$ is a transposition.

(2) Let $G=Z_2\times Z_2\times Z_2$, where Z_2 is a cyclic group of order 2. Then the mapping

$$\theta(z_1,z_2,z_3)=(z_2,z_3,z_1)$$

is an automorphism of the group G and $\theta^3=id$. Hence by Hosszú theorem the set G with the operation

$$f(x_1^7)=x_1\cdot\theta x_2\cdot\theta^2x_3\cdot x_4\cdot\theta x_5\cdot\theta^2x_6\cdot x_7$$

is a non-commutative 7-group. It is not reducible and it is σ-permutable for all $\sigma\in H$, where H is a subgroup of S_8 generated by the following transpositions: $(1,8)$, $(4,8)$ and $(7,8)$.

(3) Let $(G,+)$ be an abelian group and let

$$f(x_1^5)=x_1-x_2+x_3-x_4+x_5\,.$$

Then (G,f) is a 5-group which is σ-permutable for every $\sigma\in H$, where H is a subgroup of S_6 generated by the transpositions: $(1,3)$, $(1,5)$, $(2,4)$, $(2,6)$, $(3,5)$ and $(4,6)$.

In the sequel we shall use the following abreviations: if $\sigma\in S_{n+1}$, then always $i=\sigma^{-1}(n+1)$ and $j=\sigma(n+1)$.

2. σ-permutable n-groups

In [16] we proved the following two theorems.

Theorem 1. *Every* σ*-permutable* n*-group is medial.* □

Theorem 2. *If* $\sigma(n+1)\neq n+1$, *then an* n*-group* $(G,f)=der_{\theta,b}(G,\cdot)$ *is* σ*-permutable iff* $(G,\cdot)$ *is a commutative group,* $\theta^{\sigma(k)-k-j+1}=\omega$ *for* *all* $k \in N_n - \{i\}$ *and* $b = b^{-1}$. □

As a simple consequence we obtain

Corollary 1. *If an n-group* $(G, f) = der_{\theta,b}(G, \cdot)$ *is* σ*-permutable and* $\sigma(n+1) \neq n+1$, *then* $x^{<2>} = x$ *for* $x \in G$ *and* $\theta^{d(t,\sigma)} = \omega$ *for all* $t \in N_n - \{i\}$, *where* $d(t,\sigma) = \sigma(t) - t + i - 1$.

Proof. The condition $x^{<2>} = x$ immediately follows from the definition of σ-permutable n-groups. Since $\theta^{i+j-2} = id$, then by Theorem 2

$$\theta^{d(t,\sigma)} = \theta^{\sigma(t)-t+i-1-(i+j-2)} = \theta^{\sigma(t)-t-j+1} = \omega \,. \qquad \square$$

Let $(G, \cdot)$ be a group and let θ be its an automorphism. Let

$$id\,(G,\theta) = \{\, v \in N : \theta^v = id \,\}\,,$$

$$\omega\,(G,\theta) = \{\, t \in N : \theta^t = \omega \,\}\,.$$

From the above results it follows that for every σ-permutable n-group $(G, f) = der_{\theta,b}(G, \cdot)$ there exists

$$v_0 = min\,\{\, v \in id\,(G,\theta) : v \neq 0 \,\}$$

and

$$t_0 = min\,\{\, t \in \omega\,(G,\theta) : t \neq 0 \,\}\,.$$

Obviously $t_0 \leq v_0$ and $v_0 | v$ for every $v \in id\,(G,\theta)$.

Lemma 1. *Let* $(G, f) = der_{\theta,b}(G, \cdot)$ *be a* σ*-permutable n-group, where* $\sigma(n+1) \neq n+1$. *Then*

$$\theta^t = \omega \iff t \equiv t_0\,(mod\;v_0)\,.$$

Proof. It is clear that $t \equiv t_0\,(\,mod\,v_0)$ implies $\theta^t = \theta^{t_0} = \omega$.

Conversely, if $\theta^t = \omega$ for some $t \in N$, then $t \geq t_0$. Thus $t = st_0 + r$ for some $s \in N$ and $0 \leq r < t_0 \leq v_0$. For even s we have $\omega = \theta^t = \theta^r\theta^{st_0} = \theta^r$, which implies $r = 0$. Hence $t = st_0$. Thus $\omega = \theta^t = \theta^{st_0} = id$. This is possible only for a Boolean group, but in this case $t_0 = v_0$ and

$t \equiv t_0 \,(\,mod\, v_0)$. If s is odd, then $s = 2k+1$ and $\omega = \theta^t = \theta^{st_0}\theta^r = \omega\theta^r$, which gives $\theta^r = id$. Thus $r = 0$ because $r < v_0$. But $2kt_0 \in id(G,\theta)$ and $v_0 | 2kt_0$, then $t = (2k+1)t_0 = 2kt_0 + t_0$ implies $t \equiv t_0 \,(\,mod\, v_0)$. □

Theorem 3. *Let $\sigma \in S_{n+1}$, $\sigma(n+1) \neq n+1$ and let $(G,\cdot)$ be a commutative group. Then an n-group $(G,f) = der_{\theta,b}(G,\cdot)$ is σ-permutable iff*

(1) $\sigma^{-1}(n+1) + \sigma(n+1) \equiv 2\,(\,mod\, v_0)\,,$

(2) $d(t,\theta) \equiv t_0\,(\,mod\, v_0)$ *for every* $t \in N_n - \{\,i\,\}$.

□

Proof. From the above results it follows that every σ-permutable n-group of the form $(G,f) = der_{\theta,b}(G,\cdot)$ satisfies (1) and (2).

On the other hand, if $\sigma \in S_{n+1}$, $\sigma(n+1) \neq n+1$, $(G,\cdot)$ is a commutative group and an n-group $(G,f) = der_{\theta,b}(G,\cdot)$ satisfies (2), then for every $x \in G$ and $t \in N_n - \{\,i\,\}$, we get

$$\theta^{t-2\sigma^{-1}(t)+i}(\theta^{\sigma^{-1}(t)-1}x \cdot \theta^{i+t-2}x) = \theta^{t-\sigma^{-1}(t)+i-1}x \cdot \theta^{2(t-\sigma^{-1}(t)+i-1)}x =$$

$$\theta^{d(t,\sigma)}x \cdot \theta^{2d(t,\sigma)}x = x^{-1} \cdot x = e,$$

where e is the identity of $(G,\cdot)$. Hence

(3) $\theta^{\sigma^{-1}(t)-1}x \cdot \theta^{i+t-2}x = e$

for every $x \in G$, and, in particular

$$\theta^{\sigma^{-1}(t)-1}x_{\sigma(\sigma^{-1}(t))} \cdot \theta^{i+t-2}x_t = e$$

for every $t \in N_n - \{\,i\,\}$ and $x_t \in G$.

Putting $x = b$ in (3) we obtain $b^2 = e$, which together with (1), (2) and (3) gives

$$f(x_{\sigma(1)}^{\sigma(i-1)}, f(x_1^n), x_{\sigma(i+1)}^{\sigma(n)}) =$$

$$x_{\sigma(1)} \cdot \theta x_{\sigma(2)} \cdot \ldots \cdot \theta^{i-1}(x_1 \cdot \theta x_2 \cdot \ldots \cdot \theta^{n-1}x_n \cdot b) \cdot \theta^i x_{\sigma(i+1)} \cdot \ldots \cdot \theta^{n-1}x_{\sigma(n)} \cdot b =$$

$$(\theta^{\sigma^{-1}(1)-1}x_{\sigma(\sigma^{-1}(1))} \cdot \theta^{i-1}x_1) \cdot (\theta^{\sigma^{-1}(2)-1}x_{\sigma(\sigma^{-1}(2))} \cdot \theta^i x_2) \cdot \ldots$$

$$\ldots\cdot(\theta^{\sigma^{-1}(n)-1}x_{\sigma(\sigma^{-1}(n))}\cdot\theta^{i-1+n-1}x_n)\cdot\theta^{i-1+j-1}x_{\sigma(n+1)}\cdot b^2=$$

$$e\cdot\ldots\cdot e\cdot x_{\sigma(n+1)}\cdot e=x_{\sigma(n+1)}.$$

This proves that an n-group $(G,f)=der_{\theta,b}(G,\cdot)$ satisfying (1) and (2) is σ-permutable, which ends the proof. □

Theorem 4. *Let $\sigma\in S_{n+1}$ and $\sigma(n+1)=n+1$. If $(G,\cdot)$ is a commutative group, then an n-group $(G,f)=der_{\theta,b}(G,\cdot)$ is σ-permutable if and only if $\sigma(k)\equiv k\,(\,mod\,v_0)$ for all $k\in N_n$.*

Proof. Since $(G,\cdot)$ is commutative, then $v_0\le n-1$. It is clear that for $\sigma\in S_{n+1}$ and $k\in N_n$

$$\sigma(k)\equiv k\,(\,mod\,v_0)\iff\sigma^{-1}(k)\equiv k\,(\,mod\,v_0)\,.$$

If $\sigma\in S_{n+1}$, $\sigma(n+1)=n+1$ and $\sigma(k)\equiv k\,(\,mod\,v_0)$ for all $k\in N_n$, then $\theta^{k-1}=\theta^{\sigma(k)-1}$. Thus, by the commutativity of $(G,\cdot)$, we get

$$f(x_1^n)=x_1\cdot\theta x_2\cdot\theta^2x_3\cdot\ldots\cdot\theta^{n-1}x_n\cdot b=$$

$$\theta^{\sigma(1)-1}x_{\sigma(1)}\cdot\theta^{\sigma(2)-1}x_{\sigma(2)}\cdot\ldots\cdot\theta^{\sigma(n)-1}x_{\sigma(n)}\cdot b=$$

$$x_{\sigma(1)}\cdot\theta x_{\sigma(2)}\cdot\theta^2x_{\sigma(3)}\cdot\ldots\cdot\theta^{n-1}x_{\sigma(n)}\cdot b=f(x_{\sigma(1)}^{\sigma(n)}),$$

which proves that this n-group is σ-permutable.

Conversely, if $(G,f)=der_{\theta,b}(G,\cdot)$ is σ-permutable and $\sigma(n+1)=n+1$, then

$$x_1\cdot\theta x_2\cdot\theta^2x_3\cdot\ldots\cdot\theta^{n-1}x_n\cdot b=x_{\sigma(1)}\cdot\theta x_{\sigma(2)}\cdot\theta^2x_{\sigma(3)}\cdot\ldots\cdot\theta^{n-1}x_{\sigma(n)}\cdot b\,.$$

Putting $x_k=x$ for some $k\in N_n$ and $x_t=e$ for all $t\neq k$, we get $\theta^{k-1}(x)=\theta^{\sigma^{-1}(k)-1}(x)$. Thus $k\equiv\sigma^{-1}(k)\,(\,mod\,v_0)$, which completes our proof. □

Let (G,f) be an arbitrary n-groupoid. By $(G,f^{\#})$ we denote *the dual n-groupoid*, i.e. the set G with an n-ary operation defined by

$$f^{\#}(x_1^n)=f(x_n,x_{n-1},x_{n-2},\ldots,x_2,x_1)\,.$$

It is clear that (G,f) *is an n-group iff* $(G,f^{\#})$ *is an n-group.* Moreover, (G,f) is σ-permutable iff (G,f) is τ-permutable for some $\tau\in S_{n+1}$.

Theorem 5. *Let* $\sigma\in S_{n+1}$, $\sigma(n+1)\neq n+1$. *Then an n-group* (G,f) *is* σ*-permutable iff an n-group* $(G,f^{\#})$ *is* τ*-permutable, where* τ *is a permutation defined by the formula*

$$\tau(k)=\begin{cases} n+1-\sigma(n+1) & for\quad k=n+1\,,\\ n+1-\sigma(n+1-k) & for\quad k\in N_n-\{n+1-\sigma(n+1)\}\,,\\ n+1 & for\quad k=n+1-\sigma(n+1)\,.\end{cases}$$

Proof. It is clear that $\sigma(n+1)\neq n+1$ iff $\tau(n+1)\neq n+1$. Moreover, by Hosszú theorem and our Theorem 1 an n-group (G,f) is $<\theta,b>$-derived from a commutative group $(G,\cdot)$, where $\theta^{n-1}=id$ and $\theta b=b$.

Let $\psi=\theta^{-1}$. Then $\psi(b)=b$, $\psi^{n-1}=id$ and ψ is an automorphism of the group $(G,\cdot)$. Moreover

$$f(x_1^n)=x_1\cdot\psi^{n-2}x_2\cdot\psi^{n-3}x_3\cdot\ldots\cdot\psi^2x_{n-2}\cdot\psi x_{n-1}\cdot x_n\cdot b\,.$$

Thus, by the commutativity of $(G,\cdot)$ we obtain

$$f^{\#}(x_1^n)=f(x_n,x_{n-1},...,x_2,x_1)=x_1\cdot\psi x_2\cdot\psi^2x_3\cdot\ldots\cdot\psi^{n-2}x_{n-1}\cdot x_n\cdot b\,,$$

which proves that an n-group $(G,f^{\#})$ is $<\psi,b>$-derived from $(G,\cdot)$.

Since an n-group (G,f) is σ-permutable, $\sigma(n+1)\neq n+1$, then

$$x_{\sigma(1)}\cdot\theta x_{\sigma(2)}\cdot\ldots\cdot\theta^{i-1}(x_1\cdot\theta x_2\cdot\ldots\cdot\theta^{n-1}x_n\cdot b)\cdot\theta^ix_{\sigma(i+1)}\cdot\ldots\cdot\theta^{n-1}x_{\sigma(n)}\cdot b=x_{\sigma(n+1)}\,.$$

Replacing θ by ψ^{n-2} and putting y_{n+1-k} instead of x_k for all $k\in N_n$ we obtain

$$y_{\tau(1)}\cdot\psi y_{\tau(2)}\cdot\psi^2y_{\tau(3)}\cdot\ldots\cdot\psi^{n-i-1}y_{\tau(n-i)}\cdot\psi^{n-i}(y_1\cdot\psi y_2\cdot\psi^2y_3\cdot\ldots$$

$$\ldots\cdot\psi^{n-1}y_n\cdot b)\cdot\psi^{n-i+1}y_{\tau(n-i+2)}\cdot\ldots\cdot\psi^{n-1}y_{\tau(n)}\cdot b=y_{\tau(n+1)}\,,$$

which means that $(G,f^{\#})$ is a τ-permutable n-group.

The converse is obvious. □

Theorem 6. *Let* $\sigma\in S_{n+1}$, $\sigma(n+1)=n+1$. *Then an n-group* (G,f) *is* σ*-permutable iff an n-group* $(G,f^{\#})$ *is* τ*-permutable, where* τ *is a permutation defined by* $\tau(n+1)=n+1$ *and* $\tau(k)=n+1-\sigma(n+1-k)$ *for* *all* $k \in N_n$.

Proof. The proof is analogous to the proof of Theorem 5. □

3. Isomorphisms of σ-permutable n-groups

As is well known (cf. [5]) retracts of isomorphic n-groups are isomorphic. On the other hand, retracts of non-isomorphic n-groups also can be isomorphic. In [6] conditions on isomorphisms of retracts of n-groups were given, which imply that the polyadic groups from which the retracts are taken are isomorphic. For the case of commutative retracts (in particular, for the case of σ-permutable n-groups) these conditions have even simpler form.

For example, from Theorem 3 in [6], we obtain

Theorem 7. *Two σ-permutable n-groups $der_{\theta_A,a}(A,\cdot)$ and $der_{\theta_B,b}(B,\cdot)$ are isomorphic iff there exists an isomorphism $\phi : (A,\cdot) \to (B,\cdot)$ and an element $c \in B$ such that*

$$\phi(a) = c^2 \cdot \theta_B c \cdot \theta_B^2 c \cdot \ldots \cdot \theta_B^{n-2} c \cdot b \,,$$

and

$$\phi\theta_A(x) = \theta_B\phi(x) \ \text{ for all } x \in A \,.$$

□

On the other hand, Corollary 7 from [6] implies the following characterization of isomorphisms of σ-permutable n-groups.

Theorem 8. *Two σ-permutable n-groups (A,g) and (B,f) are isomorphic iff for an element $a \in A$ there exist an element $b \in B$ and an isomorphism $\phi : ret_a(A,g) \to ret_b(B,f)$ such that*

$$\phi g(a, x, \overset{(n-3)}{a}, \bar{a}) = f(b, \phi(x), \overset{(n-3)}{b}, \bar{b}) \ \text{ for all } x \in A$$

and

$$\phi g(a, a, ..., a) = f(b, b, ..., b) \,.$$

□

As a consequence of the above results and Corollary 2 from [16], we get

Corollary 2. *Let $(A,g) = der_{\theta_A,e_A}(A,\cdot)$ and $(B,f) = der_{\theta_B,e_B}(B,\cdot)$ be two σ-permutable n-groups. If n is even (in general, if retracts of (A,g) and (B,f) are Boolean), then n-groups (A,g) and (B,f) are isomorphic iff there exist an isomorphism $\phi : (A,\cdot) \to (B,\cdot)$ and an element $c \in B$*

such that

$$\theta_B c \cdot \theta_B^2 c \cdot \theta_B^3 \cdot \ldots \cdot \theta_B^{n-2} c = e_B$$

and

$$\phi\theta_A(x) = \theta_B\phi(x) \quad for\ all \quad x \in A.$$

□

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Abstract

An n-ary group (n-group) (G, f) is called σ-permutable, where σ is a permutation of the set $\{1, 2, ..., n+1\}$, if and only if

$$f(x_{\sigma(1)}, x_{\sigma(2)}, ..., x_{\sigma(n)}) = x_{\sigma(n+1)} \Longleftrightarrow f(x_1, x_2, ..., x_n) = x_{n+1}$$

for all $x_1, x_2, ..., x_{n+1} \in G$. Such n-groups are a generalization of several classes of n-groups considered in our previous papers. In this paper we give some examples of σ-permutable n-groups and describe properties of such groups. Necessary and sufficient conditions for an n-group to be σ-permutable are determined. We give also several conditions under which such n-groups are isomorphic.

Uwagi o permutowalnych n-grupach

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Streszczenie

Grupa n-arna (n-grupa) (G, f) nazywa się σ-prmutowalną, gdzie σ jest permutacją zbioru $\{1, 2, ..., n+1\}$, wtedy i tylko wtedy gdy

$$f(x_{\sigma(1)}, x_{\sigma(2)}, ..., x_{\sigma(n)}) = x_{\sigma(n+1)} \Longleftrightarrow f(x_1, x_2, ..., x_n) = x_{n+1}$$

dla wszystkich $x_1, x_2, ..., x_{n+1} \in G$. Takie n-grupy są uogólnieniem kilku klas n-grup rozpatrywanych w naszych poprzednich pracach. W tej pracy podajemy kilka przykładów σ-permutowalnych n-grup i opisujemy własności tych n-grup. Podajemy warunki konieczne i wystarczające na to by n-grupa była σ-permutowalna. Podajemy też warunki przy spełnieniu których takie n-grupy są izomorficzne.