

Semiotic and didactic model of mathematical expressions

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Introduction. When a teacher is preparing a mathematic lesson, he must think over the way of pupil's work to make them understand mathematical essence. Famous educationists emphasize that meaning of mathematical expression - guarantee of notion understanding - means more than only mathematical material. Following this idea I have investigated required features of expression meaning which accompany understanding its mathematical essence. Having the results I have worked up a didactic model of expressions meaning which allows to improve oral delivery of mathematical essence. In this way I have found several areas of its effective application. Moreover the research shows that this model may be used as a criterion of oral forms effectivity of mathematical essence delivery to emphasize the expression of teacher's language.

1. Subjects and task

We can say without any exaggeration that the effectiveness of the teaching and learning of mathematics depends absolutely upon the efficiency of an intercommunication. A lot of different conditions determines the teaching and learning level of the mathematics. One of such conditions in the didactic process is the performance of mathematical phrase meaning category. As the mathematical phrases are indispensable in the intercommunication during the mathematical lessons, the explanation of the essence and the function of the mathematical phrase meaning category can help to increase the knowledge and understanding the teaching and learning process itself and it allows to describe scientifically media and interaction methods for the conceptions forming process and the improvement of its effectiveness. I think therefore that the meaning of mathematical phrases is one of the basic categories of mathematical didactics.

The meaning category as central category of semiotics is very interesting for physicists, logicians, psychologists, for the informatics specialists and other representatives of never fields of knowledge. This category is

often the subject of research within particular sciences. I am interested too in this category as it is an important knowledge theory problem, having application in the mathematical education field, namely as a problem that solution will be helpful in the improvement of the intercommunication efficiency in mathematical teaching and learning. That is as follows:

- I am not going to talk here about a mathematical concept. It belongs to psychology. It can be indeed defined in many ways. I understand it here in the following way: A mathematics concept is a reflection of an abstract concept in a persons thought.

- I am not going to talk here about a mathematical expression too, because a mathematics expression has been defined in foundations of mathematics very exact.

- I am considering development of a mathematical concept as a phrasing of a mathematical concept with mathematical expressions. The subject's task was to construct a mathematical model for this process.

2. Results

3.1. The semiotic model of the mathematical phrases meaning My researchs of the literature with respect to the topic of meaning in the range of the particular sciences has proved that there are a lot of different views upon the meaning matter. More than once those differences result from the peculiarity of the cultivated scientifics and the purpose in which we are using the concept of meaning. Consideration for the part of phrase meaning in the intercommunication it caused the attempts to unit all arguments enriching our knowledge about intercommunication. Applying the analitical-synthetic construction method I obtained semiotic model of the sing meaning. I described a meaning as a system uniting unpeculiarity and unaccidental attributes of meaning which are examined in framework of particular sciences. I present my idea as system of four elements as follows:

(a) The elements of the system:

O - recognizing subject **R** - recognized object **Z** - phrase **P** - conception.

(b) The basic relation between the elements (they aren't the relation in the sense of the set-theory, they have other attributes):

(OR) - **O** recognizes indirectly **R** (in the recognizable, uninstrumental process), **(RO)** - **R** influences **O** (directly or indirectly, it isn't the

contrary relation to (OR)), (RP) - **Objects R reflects himself in consciousness O as notion** (it's the composed relation), (ZP) - **consists in the mutual reference of a sing and conception about a recognized object**, (OZ) - the relation of a sing perception by O (ZO) - consist in an influence on the nervous system of a person (it isn't the contrary relation to (OZ)), (OP) - **O develops a conception P** (it is the fundamental relation of the meaning), (PO) - **P influences the understanding of meaning by person O** (it isn't the contrary relation to (OP)), (ZR) - **Z refers to R** is a representative function of the sing.

All the elements and the simple relation are in a dialectic unity, that is a main attribute in quality of the meaning system deciding about the unreduction of meaning system for any of its components or their sets. Each reduction causes an impoverishment of meaning, an one-sidedness of understanding and even a vitiation of understanding. The system performs not only inside but outside as well, when its elements unite with the elements of other meaning systems. (c) The effectiveness of the intercommunication in the teaching and learning mathematics depends on a many-sidedness of the meaning system function. The perception of relations which I have called *united*, helps to understand what is the complexity and many-sidedness of the system function both inside and outside system. I distinguished two their kinds: single and with repetitions, as follows:

Single: (OZR), (ORZ), (OZRP), (OPZR), (ORPZ),

With repetitions: (OZPOR), (ORPOZ), (OZPOPR), (ORPOPZ).

Each relation has its own qualitative characteristic. Taking into account even only a partly wealth of an outside system working, one can notice that the united relations come together to the families determining different aspects of phrases meaning. I have distinguished in this way six aspects of meaning, i.e. the didactic model of the mathematical phrases meaning, as all six aspects together, as unity of these aspects.

3.2. The didactic model of the mathematical phrases meaning In the suggested here didactic model of meaning of mathematical expressions I consider the following aspects:

1' objective aspect (It answers the question: *what is it ?*), 2' algorithmic aspect (It answers the question *how to identify this object?*), 3' perspective

aspect (This object is towards a goal), 4' pragmatic aspect (It is a place of the object in the system of relations, between other concepts), 5' psychologic aspect (It is a weight of the object in the system of relations, it is motivation too, etc.) 6' functional aspect (It is some spheres of mathematical activity).

The first two of those aspects are measurable, there are the exact aspects. The others depend on many variables and practically are not measurable. But one can not reject them. It is those unmeasurable aspects that are source of mathematical activities. One needs to consider all six in order to raise the effectiveness of teaching.

3. Conclusions and implications

In the employment of the didactical model of the mathematical phrases meaning were the following features found. The verification of the constructed model in the teaching and learning practice has proved that the didactic model of mathematical phrase meaning consist only of five aspects: objective, algorithmic, pragmatic, psychologic and functional. The verification of the model has proved too, perspective aspect keeps usually as potential. The didactic model of meaning has been proved to be very useful for a research of many problems of mathematical didactics. The above model has a wide use as teaching as such is strictly related to language:

- constructing of mathematical problems, - constructing of mathematical tests, - defining of the concepts, - editing of the mathematical texts, - inventing of the teaching aids, - working out of the teaching strategies, - analysis of the causes of the difficulties and failures of the pupils.

I have ascertained that it acts as criterion for the analyses of an expressiveness and suggestiveness of the mathematical texts too.

References

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1. Syntax and semantics of the internal logic of program.

We denote by \mathbb{N} the set of all positive integers.

Let $\mathcal{A} = \{a_1, \dots, a_n\} \subseteq \mathbb{N}$ be the denumerable set of propositional variables. We write p, q and r instead of a_1, a_2 and a_3 respectively. As metavariables for elements of \mathcal{A} we use letters a and b . (Always when we say that some letters are metavariables for elements of \mathcal{A} we mean that these variables vary through this set and that we also their primed or primed variants of these letters, just as well as variables over the same set. However, no letter is used as a variable for a given set unless specifically indicated to this way. Only a and b (and $a_1, b_1, a_2, b_2, \dots, a_n, b_n, \dots$ etc.) range over \mathcal{A} and not, e.g., c, d or a (conjunction with a notion of the syntactic variable in [2]).

Definition 1.1.

The set S of propositional formulae of internal logic is the smallest set for which three following conditions hold:

- (a) $\mathcal{A} \subseteq S$
- (b) If $A \in S$ then $(\neg A) \in S$

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Abstract

In this study was research the model of the meaning in the mathematics teaching. This model was analysed and the results are described. It was clear that the meaning is complicated system of the complementary features. In the suggested here didactic model of meaning of mathematical expression consider six aspects. The above model has a wide use as teaching as such is strictly related to language.

3. Conclusions and implications

In the employment of the didactical model of the mathematical phrase meaning were the following features found: the verification of the constructed model in the teaching and learning practice has proved that the didactic model of mathematical phrase meaning consist only of six complementary aspects: linguistic, pragmatic, psychological and functional. The verification of the model has proved the comparative aspect is not usually as potential. The didactic model of meaning has been found to be very useful for a research of many problems of mathematical didactics. The above model has a wide use as teaching as such is strictly related to language.

reconstruction of mathematical problems - reconstructing of mathematical text - defining of the concepts - meaning of the mathematical text - character of the teaching aids - character of the teaching strategies - analysis of the causes of the difficulties and failures of the pupils.

I have ascertained that this investigation for the teaching of an expression can be used as a significant tool in the didactical theory.

References

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