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USING GRAPHIC DISPLAY CALCULATOR IN SOLVING SOME PROBLEMS WITH POLYNOMIALS

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Abstract

A graphic display calculator (GDC) is becoming more and more popular in teaching mathematics as it is used to examine some mathematical activities of students of almost all ages. Various modes of GDC are considered to be a useful tool in understanding of particular parts of mathematics. In most cases the properties of functions are examined by observation of their graphs. However, there are some properties of the functions which one cannot see during the graphs analysis (for example properties concerning complex roots of polynomials). The aim of this paper is to analyse how 17-and-18-year-old students for whom GDC is an obligatory device can generalize some relations between polynomials and so called "shadows" of these functions. The whole paper is concerned with in investigation of properties of quadratic, cubic and quartic functions with both real and complex roots.

1. Introduction

A graphic display calculator (GDC) has become more and more popular in the process of learning and teaching mathematics. There are lots of researches about an effective usage of this portable device (for instance [1], [2], [3]). Some of them show how students use GDC as a routine activity for them (for instance [4], [5]) other show teacher's expectations of using this tool (for instance [6], [13]). However, there is no lack of papers which show difficulties in using GDC (for instance [14]). In Polish school programs graphic calculator is not popular. Moreover any calculator, except for simple one (four-operations one), is forbidden during Polish exams on each level in both the middle and high schools. The unique school programme admitted by Polish Minister of Education – so called International Baccalaureate Diploma Programme – accepts the use of GDC as a mandatory device during the process of learning – teaching mathematics and during the public examinations. It is alternative programme in high schools and

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it is intended for students aged 16-19. (For further information about this program one can go to [11], [12], or websites). As using GDC is not present among Polish maths teachers. In [7], [8], [9], [10] I described my research with graphic calculator was carried out among my students attending International Baccalaureate Diploma Program class (shortly IB class). For my research I mainly chose tasks or parts of tasks intended for using IT (especially graphic calculator). However, other tasks I also used. Students solved the tasks during the normal lesson time or had comfortable conditions – they worked prior to the lesson time, individually without time limitation, having access to GDC, the Internet and computer software all the time. Unfortunately, we have no chance of knowing how the quality of conditions during solving the tasks influenced students' work (activity performed on GDC and computer was not recorded).

The main role in this paper plays the task patterned on a task proposed in "Portfolio tasks for use in 2012 and 2013 published by International Baccalaureate Organization" (more about Portfolio one can find in [11], [12]).

The current task which analysis will be considered in this paper concentrates on the investigation of some properties of polynomials.

As authors emphasized in [14] a graph of function is a crucial weapon in the mathematics learning. As early as possible students learn to recognize the important features of graphs of functions. They are taught how to find intercepts, roots, monotonicity of such functions like linear and quadratic functions. Yet, when GDC is not known or available the numbers of graphs that students can draw is rather limited. As a consequence students usually have a problem with finding or changing the scale if they use only paper and pencil for this purpose. If they have to find some properties of graphs they usually sketch graphs of "nice" functions, for instance with roots or vertices which are integers. However only after students use any technologies for such tasks they become real researchers. They can investigate many examples with different properties (different kinds and numbers of roots, etc.) Proposed task in this paper has some signs of the process of generalization. This process is considered by me in [10], where I examined so called visual templates (name proposed by Rivera in [15]). Learning on my research and remarks made in [15] I proposed the scheme presenting the process of generalization using such a special kind of tasks.

This scheme was verified in [7]. Below the scheme is quoted, (see Table 1).

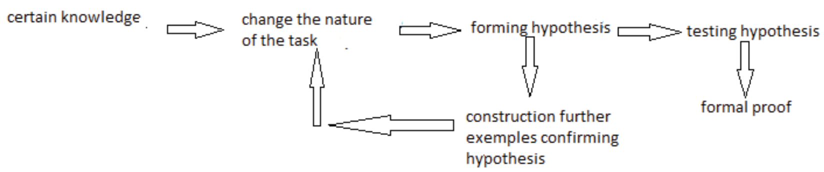


Table 1. The scheme of process of generalization proposed in [10]

In paper [3], which is worth mentioning here, one can find division of applications of GDC during almost all mathematical activities.

1. Computational Tool (evaluating a numeral expressions, estimating and rounding)
2. Transformational Tool (changing the nature of the task)
3. Data Collection and Analysis Tool (gathering data, controlling phenomena, finding patterns)
4. Visualizing Tool (finding symbolic functions, displaying data, interpreting data, solving equations)
5. Checking Tool (confirming conjectures, understanding multiple symbolic forms)

The main questions which I asked prior to the research were

1. How useful can a graphic calculator be in solving the task with different kinds of polynomials?
2. Can students observe common properties of roots using only a graphic calculator?
3. Can students solve such tasks without any technologies?

2. Main results

The research was conducted in the class with International Baccalaureate Diploma Programme. All students who took part in the research (ten 17-18-year-old students (eight boys and two girls) were taught by me. The whole research was concerned with one task patterned on the portfolio tasks for mathematics in IB programme for use in 2012-2013 titled "Shadow functions" published by International Baccalaureate Organization (IBO). Below one can find the text of the task.

The task

Consider the quadratic function $y_1 = (x-a)^2 + b^2$. *Write down the coordinates of the vertex and show that all roots of* y_1 *have the form* $a \pm ib$ *where* $i^2 = -1$. *Consider the function* y_2 *which has opposite concavity to* y_1. *Such function is so called "shadow function". Use various values for* a, b *to* *generate pairs of shadow functions. Let y_m be a line of reflection of y_1 and y_2. Find equations for y_1 and y_m. Express y_2 in terms of y_1 and y_m. On the diagram show how zeros from y_2 may be helpful in determination of roots of y_1. Consider the function $y_1 = (x+2)(x-(3+2i))(x-(3-2i))$. Let y_2 be a shadow function for y_1 which has one common zero at point -2. Repeat all points above mentioned. Check whether your results can be applied to quartics.*

This task is concerned with the observation of pairs of functions (named shadow functions) which have different roots but the graphs have reflected shapes in some lines. This task seems to be created for using GDC because students have opportunity to examine not only polynomials with real roots but also with complex roots.

One should know that prior to this research students were taught about roots of polynomials and about complex numbers Nevertheless, they did not solve such formulated tasks and did not generalize similar problems.

The research was divided in three parts. The first one was the observation of students working on the task during three consecutive 45-minute lessons. Because the task consisted of three similar parts I expected students to solve at least the first part of it (about the quadratic functions). At the beginning of the lesson the students were given the task and graphic display calculators (one copy of the task and one graphic calculator per student) and explained very carefully the problem included in the task. However, at this point the students did not obtain any particular hints. During the lesson time students worked with the task individually and wrote their solutions on the provided sheets of paper. Throughout the task I only observed students. When the time was over I gathered the sheets in order to analyse them.

The second part was the interview which was recorded. The students were asked individually the same five questions quoted below

1. How did you use GDC for solving this task (which mode of GDC did you use and why)?

2. Does GDC is a sufficient device to solve this task. What else do you need?

3. Did GDC let you generalize the problem from the task? How?

4. Can you solve this task without using any technologies?

5. Having GDC would you like to formulate any kind of task for your peers? What kind of task would it be?

In the third part of this research I provided the students the same task and made them to solve it individually during 10-day-period as their homework. After this time I gathered students final works in order to analyse them.

The analysis of students' work

As each part is considered to be a different part of an activity I will analyse them separately.

The first part concentrates on the analysis of students notes made during three consecutive lessons. Generally, students solved only the first part of the task (about the quadratic functions) because the time was limited and did not allow them to do as many attempts of examinations of similar examples as were needed. Despite the limited time, some students worked quicker and tried to check obtained generalization for cubic functions. However, they did not finish their work, so the analysis of the cubic functions is limited. It is important to notice that six students used GDC but others did not do it. Students who used GDC usually used it for sketching graphs of functions given in the task and other similar examples produced by them. Moreover, they checked other properties of the graphs (x- and y-intercepts, roots, etc.) As a result using mentioned above properties they draw some relations between roots of both functions (functions and their shadows). Below the original work of three students is presented below, in which they included generalizations between roots of y_1 and y_2 (compare the text of the task).

Student 1 found general patterns for roots of y_1 using adequate patterns for given quadratic function. Next he wrote the pattern for y_2 and examined three examples for different values of a and b, but only for a, b being integers. In the first one he found roots for both y_1 and y_2 using adequate patterns but in further examples he found roots using GDC. He finally wrote conclusion but only for the first example proposed by him, i.e. for $a = 2$ and $b = 1$. What is important, he did not checked whether his conclusion was suitable for further examples. Below the original piece of work of student 1 is provided, (see Table 2).

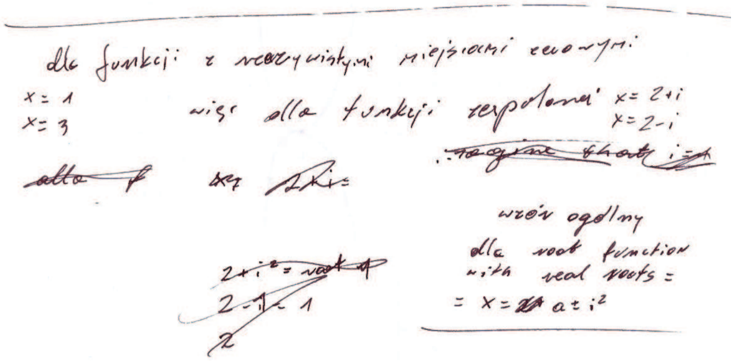


Table 2. A part of original student's work (student 1)

for the function with real roots $x = 1, x = 3$, so for complex functions $x = 2+i, x = 2-i$. General term for root function with real roots $x = a \pm i^2$ (trans. by J. Jureczko).

This pattern cannot be recognized as general pattern. The student should have checked his pattern for other examples but he did not do it.

According to the task he gave the pattern for y_m and expressed y_2 in terms of y_1 and y_m using only general patterns for y_1 and y_2 and solving the following system of equations

$$\begin{cases} y_1 = (x+a)^2 + b^2 \\ y_2 = -(x+a)^2 + b^2 \end{cases}$$

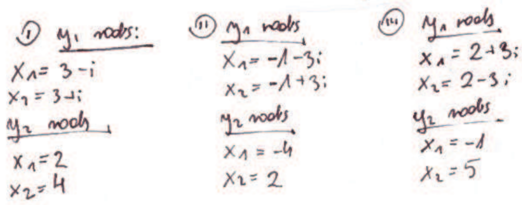
where $y_m = b^2$.

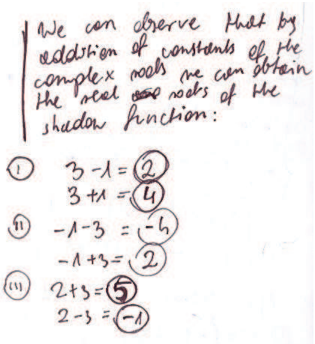
Following the instructions student 2 did not find roots of y_1 but only checked that they really were by substitution consequently obtaining tautology. She found the pattern for y_2 properly but did not formulate the general pattern (only used in investigated examples). Then she examined three examples for different values of a and b, in each case checking algebraically whether complex roots were really roots of given function. Yet, for roots of y_2 she did not do it. For all examples she found roots using GDC.

Following the further points of the task she found the pattern for y_m. Nevertheless, in order to express y_2 using only terms y_1 and y_m she incorrectly assumed that $y_2 = -y_1$ and then she wrote the pattern for $-y_1$ twice and crossed it. In spite of this mistake she used proper patterns for y_1 and y_2. By writing in the next line

$$(x-a)^2 + y_m = -(-(x-a)^2 + y_m) + 2y_m$$

and by reducing the similar terms she obtained $y_2 = -y_1 + 2y_m$ which contradicted her assumptions. Additionally, she did not comment on this situation. Afterwards it she tried to find the relations between roots of both quadratic functions. The first example if for $a = 3, b = -1$, the second one is for $a = -1, b = -3$ and the last one is for $a = 2, b = 3$. Below one can find the part providing the conclusion, (see Table 3.)




We can observe that by addition of constants of the complex roots we can obtain the real roots of the shadow function:

(i) 3−1 = 2
3+1 = 4

(ii) −1−3 = −4
−1+3 = 2

(iii) 2+3 = 5
2−3 = −1

Table 3. A part of original student's work (student 2)

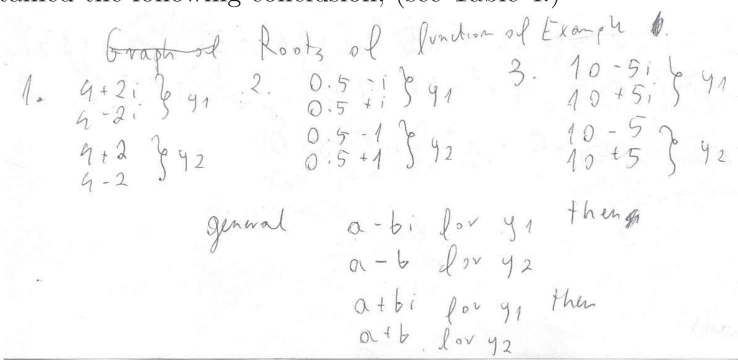
In comparison to student 1 student 2 did not write the general pattern algebraically. She only wrote verbally how one could obtain roots for arbitrary shadow functions. Moreover, she did not examine further examples to confirm her conclusion. Even though she tried to use her conclusions shown above for given cubic function, she did not check it for any examples of the cubic function.

Student 3 found the roots of y_1 using the same method as student 1 and gave the pattern for y_2 properly. Then he examined three examples for different pairs of values of a and b by finding roots using paper-pencil method and checking with GDC. Although he used proper patterns for y_1 and y_2 he concluded general expression for y_2 in the form

$$y_2 = -(y_1 - y_m)^2 + y_m.$$

He made a mistake as he substituted the expression for $(x - a)$ by $y_1 - y_m$ omitting the square power in the first expression.

For the values: $(a = 4, b = 2)$, $(a = 0.5, b = 1)$ and $(a = 10, b = 5)$ he obtained the following conclusion, (see Table 4.)


~~Graph of~~ Roots of function of Example

1. 4+2i, 4−2i } y_1; 4+2, 4−2 } y_2
2. 0.5−i, 0.5+i } y_1; 0.5−1, 0.5+1 } y_2
3. 10−5i, 10+5i } y_1; 10−5, 10+5 } y_2

general a−bi for y_1 then
a−b for y_2
a+bi for y_1 then
a+b for y_2

Table 4. A part of original student's work (student 3)

Student 3 provided a general pattern for roots of shadow functions algebraically. Similarly to student 1 and student 2 he did not check his general pattern by examining further examples.

As student 2 he tried to apply his conclusion for given cubic function. Yet, he did not check it for any cubic function.

Three other students tried to obtain the similar results. Although they had a good idea, the time probably did not allow them to finish them this part of the task. Other students, who did not use GDC in solving the task, operated only on the general patterns (using letters instead of numerical examples of functions), but none of them obtained neither relations between the graphs showing on the task nor other properties mentioned in the task.

The next part of the research was an interview with participants which was done immediately after these three lessons. Below there are provided citations of only those three students whose solutions were analysed above (the transcript from Polish is translated by me)

Answers for question 1:

1. Using it I could find roots faster. I could sketch graphs and check different kinds of functions. (I used) Graph, Equation, Run Math.

2. GDC helped me in sketching graphs which was needed to solve the task and to obtain roots. It also made the task easier and helped me to get a solution faster. (I used) Graph.

3. I checked my answers whether they were proper and sketched graphs of functions I tried to compare this functions with complex numbers but I did not manage to do it because the time was too short.

Comments: Students claimed that they used GDC generally as a visualizing and checking tool (compare [3])

Answers for question 2:

1. This device is sufficient to solve this task because it has all needed options to do it.

2. I think this device suffices but if I want to prepare it better I would need Geogebra to sketch graphs.

3. If this task was precisely done I would need to use computer to sketch graphs.

Comments: Students claimed that they did not need any other devices as far as they had to prepare their work very carefully. They understood that the screen of GDC an its software was as not precise as computer software.

Answers for question 3:

1. GDC helped me to generalize the problem posted in the task and helped to make conclusions

2. No, rather not.

3. Rather not, I wrote on the piece of paper and used GDC only for calculations and graphs.

Comments: Students claimed that GDC helped them only for checking a few examples (as a transformational tool), but for generalization human logical thinking is needed. As a consequence only student 1 gave positive answer, but he did not obtain generalization required in the task properly.

Answers for question 4:

1. I would be able to solve this task without using GDC but it would take me more time.

2. I think so, but it would be more workable.

3. It would take more time.

Comments: Students unanimously claimed that they were able to solve the task without any technologies but it would take more time.

Answers for question 5:

1. No, I cannot formulate any tasks because I have no idea how to do it.

2. If I created some task it would be the task of type with functions because GDC is more useful in sketching graphs and finding their properties.

3. I would try, it would be interesting.

Comments: Students had no precise idea how to generate the new task probably because they did not have enough experience (this task was the first task of this type for them).

The last part of the research concentrate on the analysis students' work which was done during 10-day period as their homework (without time limitations and with the full access to GDC and computer software).

During the 10-day-period almost all students solved the task correctly, the method of thinking which started during the lesson time was continued by them. What is worth emphasizing all students used GDC and graphic computer software to solve the task. Although this part of the research was the most progressive as the students solved the task without time limitation and with comfortable conditions the researcher does not know about anything about the attempts of solving the task, the process of reasoning, methods of work or even the time needed for solving the task. Without any students' explanations it is difficult to analyse the process of solving the task.

3. Final remarks

To summarize that students solving the task worked under two different conditions. The first one was during the three consecutive lessons which were during one day. In this part of research time for solving the task was limited. Students who used GDC for this task mostly obtained a part of required solution (especially for quadratic function).

What is important to notice: the parts where they used GDC were without any mistakes, but when students had to conclude some general patterns they made mistakes in reasoning (student 2 and 3) and other calculations (student 3). If they had worked without any technology they probably would have made more mistakes and the general statement about roots of functions would be impossible. What is strange students, for producing examples of functions, used only integers or simple fractions instead of constants a and b. Some students (student 3) preferred to calculate roots without GDC and only checked the result. Students probably were afraid of not obtaining a precise solution and they were right because for other constants they could not perceive required relations.

Generally, students used GDC for sketching graphs, checking results and some simple calculations. As was mentioned in paper [3] they used it in almost all way except for "Data Collection and Analysis Tool", but they did not use it during learning, so this may be the reason for this situation.

What is worth pointing out students in this task omitted two steps proposed in my scheme. After making hyphotesis they did not try to do any further examples to confirm it. It followed some dangers of incorrect general pattern (see work of student 1). They did not do any formal proof for confirming their generalized patterns although they were able to do it.(It was mentioned in the paper [10] too).

After analysis of the research, one can make the conclusion that the role of GDC is double: to form hypothesis (by examining a big number of similar examples made using GDC) and to formulate the general patterns.

In spite of some problems with using GDC it is still worth analysing students' work, especially in order to find the role of GDC in process of working on more complicated tasks, because if one has GDC or other computer software one can draw a graph of each function, check its main properties even if the roots are not integers or rationals. If a student has technological devices one can make a number of attempts in short time which make it easier to observe similarities or differences and conclude generalizations of observed objects.

References

[1] J. Berry, E. Graham, A. Smith, *Observing students working styles when using graphic calculators to solve mathematics problems*, International Journal of Mathematical Education in Science and Technology, **37(3)** (2006), 291-308.

[2] R. G. Brown, *Does the introduction of the graphic display calculator into system-wide examinations lead to change in the types of mathematical skills tested?*, Educational Studies in Mathematics, **73(2)** (2010), 181-203.

[3] H. M. Doerr, R. Zangor, *Creating meaning for and with the graphing calculator*, Educational Studies in Mathematics, **41** (2000), 143-163.

[4] P. A. Foster, P. C. Taylor, *An investigation of communicative competence an upper-secondary class where using graphics calculators was routine*, Educational Studies in Mathematics, **52** (2003), 57-77.

[5] P. A Foster, U. Mueller, D. Haimes, J Malone, *Impact on school assessment where use of graphics calculators is mandatory in a related public examination*, Int. J. Math. Educ. Sci. Technol., **34(3)** (2003), 343-359.

[6] E. Graham, C. Headlam, J. Sharp, B. Watson, *An investigation into whether student use of graphics calculators matches their teacher's expectations*, International Journal of Mathematical Education in Science and Technology, **39(2)** (2008), 179-196.

[7] J. Jureczko, *The role of The Graphic Display Calculator in forming conjectures for instance a special kind system of linear equations*, Acta Mathematica **17** (2014), 69-74.

[8] J. Jureczko, *The use of graphic display calculator and computer software in mathematical modelling*, in: Communication in the mathematical classroom, ed. M. Pytlak, (2014), 134-144.

[9] J. Jureczko, *The role of the graphic display calculator in learning mathematics on the basis of international baccalaureate students' experiences (in Polish)*, Współczesne problemy nauczania matematyki, (2013).

[10] J. Jureczko, *The process of generalization of 17-19 aged students on the basis of visual templates tasks* (in Polish), in Współczesne problemy nauczania matematyki, red. H. Kąkol, (2013).

[11] J. Jureczko, *The role of mathematics in programs: International Baccalaureate Diploma and Polish Diploma, part 1 (in Polish)*, Nauczyciele i Matematyka plus Technologia Informatyczna, **82** Summer (2012), 26-30.

[12] J. Jureczko, *The role of mathematics in programs: International Baccalaureate Diploma and Polish Diploma, part 2* (in Polish), Nauczyciele i Matematyka plus Technologia Informatyczna, **83** (Autumn 2012), (20-26).

[13] J. J. A. Lee, D. K. McDougall, *Secondary school teachers' conceptions and their teaching practices using graphing calculators*, International Journal of Mathematical Education in Science and Technology, **41(7)** (2010), 857-872.

[14] M. Mitchelmore, M. Cavanagh *Students' difficulties in operating a graphics calculator*, Mathematics Education Research Journal, **12(3)** (2000), 254-268.

[15] F. D. Rivera, *Visual templates in patterns generalization activity*, Educ. Stud. Math., **73** (2010), 297-328.

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