EYES, OPTICS AND IMAGING: MATHEMATICS AND ENGINEERING INNOVATIONS INSPIRED BY NATURE

RASTISLAV TELGÁRSKY

Abstract

This report is about mathematics and engineering innovations inspired by the nature, in particular, about the optical devices and imaging technologies inspired by eyes of living organisms. Instead of reviewing existing mathematical methods, the author explains visual sensors in nature and also the corresponding engineering devices for further inspiration of mathematicians. *Contents:* Introduction. Eyes. Optics and Imaging. Camouflage. References. Figure Credits.

1. Introduction

The effort of mathematicians in 21st century can be divided into mathematics applied in engineering and sciences, and into abstract mathematics, which has no immediate connection to any application. The research in biology brings amazing discoveries about complexity and functioning of living organisms from viruses, bacteria, and simple organisms to complex bodies and functions of mammals. In all new discoveries the nature offers problem-solving ideas inspiring human creativity, in particular, in engineering and new technologies for vision sensors. Thus this paper focuses on eyes, optics, and imaging, which inspires mathematical modelling and engineering.

There are hundreds of books about the topics of this paper. Here are just few I have selected as complementary to this paper: Adam [1], Barrow [2], Dawkins [3], Henderson [4], Nowak [5], Tegmark [6], and Yahya [7], [8].

2. Eyes

Earthworms have no eyes, but they do possess light sensitive organs – the receptor cells. The cyclops, also called waterflies, have only one eye. Fish, birds, mammals, and many other species have two eyes. Each eye has lens

● *Rastislav Telgársky* — e-mail: rtelgarsky@cnm.edu
Central New Mexico Community College, Albuquerque, NM 87106, U.S.A.

with variable focusing, retina and many neural connections to the brain. Some of these species have eyes on sides of the head so that they get large spherical angle of vision, while others have both eyes in front of the face to provide the stereo vision with field depth, and good estimation of sizes and distances. Notostracans Triops (tadpole shrimps) have 3 eyes: 2 compound eyes plus the larval nauplius eye.

Honey bees and bumble bees have 5 eyes: 2 compound eyes, each composed of 4500 "little eyes" called ommatidia (Wikipedia: Ommatidium), and 3 simple eyes, called ocelli (Wikipedia: Simple eye in invertebrates) on top of the head (Figure 1). Honey bees can see in ultraviolet light. Their vision is trichromatic like ours, but their visible spectrum is shifted toward the ultraviolet (westmtnapiary.com/Bees_and_color.html). The sand spider

(A) Male Carpenter Bee   (B) Honey Bee Face

FIGURE 1. 5 eyes of bees: 2 compound and 3 ocelli

Sicarius Hahni has 6 eyes. The *jumping spiders*, and many other spiders, have 8 eyes (Figure 2). The eye locations have various configurations, but typically 4 eyes are on the face and form very sharp images, while the other 4 eyes are mostly motion detectors. The *starfish* has 5, 21 or 40 arms (legs), and has an eye at the end of each arm (Figure 3). Each eye is sensitive to light, but does not form an image. A single *scallop* (Figure 4) can possess over a hundred eyes! Each eye has a lens and a retina which is attached to a branch of the optic nerve (Figure 5). The *giant clam* of the South Pacific, Tridachna gigas, has over 1000 eyes, but these eyes are relatively simple *photoreceptors*.

The *housefly*, Musca domestica, has two *compound eyes* (Figure 6), called ommatidia, capable of tracking movements up to five times faster than our own eyes!

(A) Hogna Wolf Spider   (B) Jumping Spider

FIGURE 2. Eye arrangements of 8-eye spiders

(A) Starfish with 11 arms   (B) Crown of Thorns

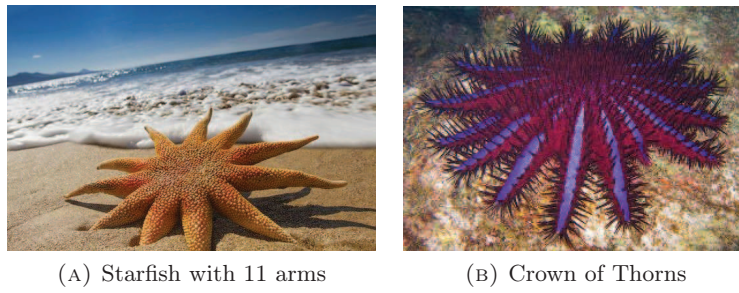FIGURE 3. Starfish

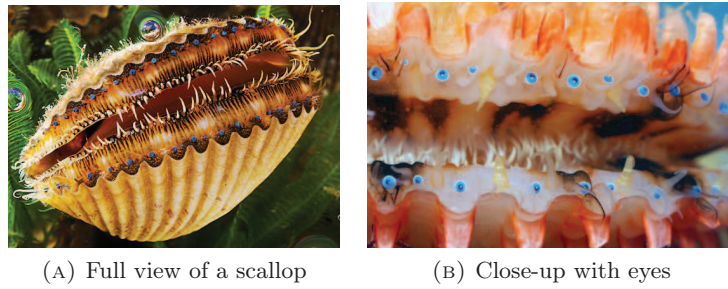(A) Full view of a scallop   (B) Close-up with eyes

FIGURE 4. Scallop with hundreds of eyes

The *fruit flies* have very small compound apposition eyes that require neural processing. They can sense rapid motion approaching 200 cycles per second. Considerable image processing is confined to a very small space.

The fish called *Stoplight Loosejaw* (Malacosteus niger) has two *biolumi-nescence* spots, one emitting a low frequency red light, and the other bright

(A) Few blue eyes   (B) Single blue eye

FIGURE 5. Close-up to scallop eyes
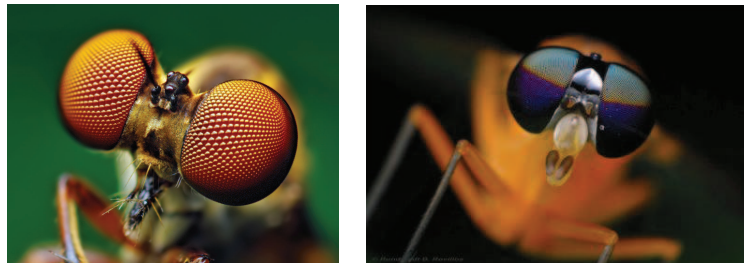
(A) Robber fly   (B) 360° view

FIGURE 6. Compound eyes of flies

green or yellow light (Figure 7). The illumination of its surrounding with
the red light allows it to see around but be invisible, because other creatures
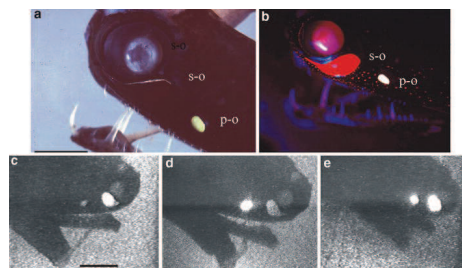cannot detect this light. When close to a prey, the fish lights up the bright

FIGURE 7. Stoplight Loosejaw uses bioluminescence from 2 spots

bioluminescence spot to see the details before catching the prey. Another

feature is the loose jaw, that is, the low jaw without the bottom allowing fast closing the mouth with sharp teeth.

3. OPTICS AND IMAGING

A drop of water on a leaf works like a natural magnifying lens, but is has a considerable distortion (Figure 8).

FIGURE 8. Water drops on a leaf

The lenses cut precisely from a glass have more desirable properties. The purpose of lenses is the magnification of images of objects, with an exception of a door viewer, which minifies the images, and provides a much wider angle of view.

The *microscope* was invented by Antonie van Leeuwenhoek (Figure 9A), who immediately discovered a gallery of small living organisms. The single lens was later replaced by an array of lenses. An Olympus objective for a microscope is shown in Figure 9B for a comparison.

(A) Leeuwenhoek device   (B) Olympus objective

FIGURE 9. Comparison of microscope's lenses

The research in chemistry, geology, biology and medicine is unthinkable without microscopes. Here is a modern microscope, shown in Figure 10, connected to a sensitive digital camera and a large digital monitor.

FIGURE 10. Modern microscope with camera and monitor

Another breakthrough in science was the invention of *telescope* by Hans Lipperhey, a German spectacle maker, who combined curved lenses to magnify objects by up to 3 times. He received the patent in 1608. Galileo Galilei heard news of the telescope, and constructed his own version of it without ever seeing one. Instead of the initial 3 power magnification, he crafted a series of lenses that in combination allowed him to magnify things by 8, 20 and eventually 30 times (Figure 11). He soon discovered the rings of Saturn.
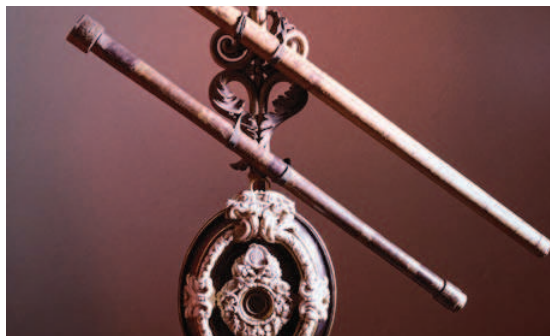

FIGURE 11. Galileo's telescope

From telescopes for visible lights the development went to infrared, ultraviolet, X-ray and radio waves telescopes. The infrared light better penetrates the earth atmosphere, but still, the earth atmosphere acts like liquid lenses and distorts the images from space. Therefore a technique was developed to detect and correct the shifts of phases in image elements. This

is called the *Adaptive Optics System* (Figure 12). The result of the phase
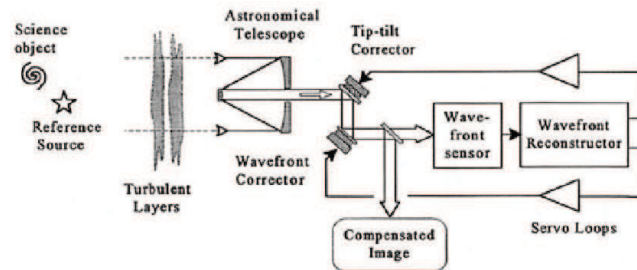

FIGURE 12. Astronomical Adaptive Optics System

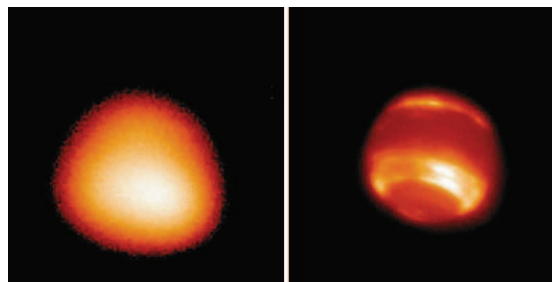correction is seen in the infrared image of Neptune (Figure 13).

FIGURE 13. Infrared image of Neptune – without and with AO

Another improvement of image quality is achieved by placing telescopes on tops of high mountains, or to the orbit of the earth, like the *Hubble Space Telescope* (Figure 14A), to the orbit of the sun, or even farther in cosmic space. However, for many astronomy hobbyists and enthusiasts, the Meade telescope ETX70A with 882 tripod and software (Figure 14B) is of very high quality, and even it has the star/planet tracking system, called *sidereal time*. So, a large number of people have astronomical observatories in back yard or front yard of their houses, or they take trips out of town where the air is cleaner for the exploration of the night sky.

The energy of electromagnetic waves coming from the space, except from the sun, is very small. So, large arrays of telescopes or radiotelescopes are built and connected, so that the compound image is constructed by computer algorithms. With larger energy per received bandwidth, the algorithms can even calculate the spectra of incoming light or radio waves. Analysis of these spectra allows for remote chemical analysis of the sources.
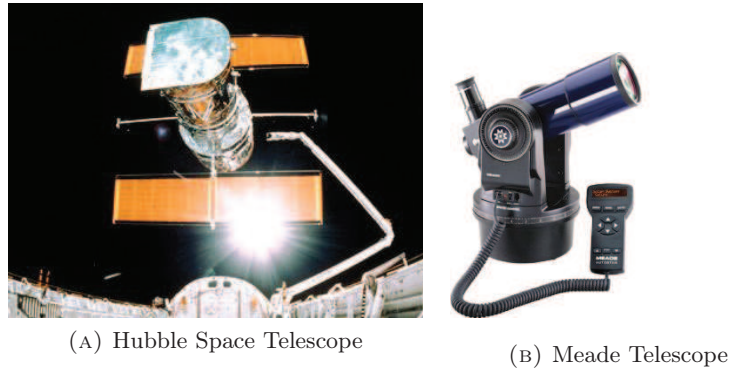
(A) Hubble Space Telescope

(B) Meade Telescope

FIGURE 14. Large and small telescopes

In particular, there is a fascinating possibility of discovering a life on an earth-like planet to a sun-like star somewhere far from our solar system, or at least, the physical and chemical conditions that may support the life forms.

In much lower frequencies, which propagate through the air, *bats* (Figure 15A) generate the *ultrasound* and reconstruct the range-Doppler images from its reflections. This well supplements their vision, and enables them to navigate and catch insects at night (Figure 15B).
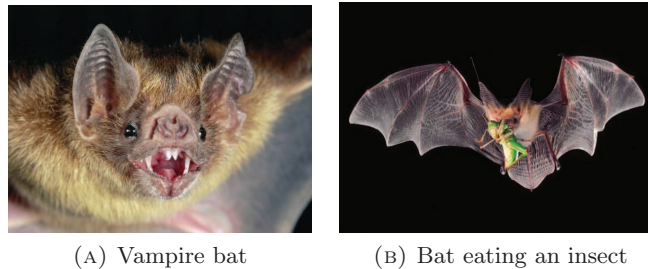

(A) Vampire bat

(B) Bat eating an insect

FIGURE 15. Bats

The *ultrasound imaging systems* used in the medical diagnostic devices (Wikipedia: Medical ultrasonography) are very sophisticated extensions of bat's image processing, although the principle is the same: the ultrasound pulses are sent, reflected, received, and processed into images. The system consists of a cylindrical hand-held scanning device containing the ultrasound transmitter and receiver, which is connected to the computer processing unit

with a real-time display and a real-time recording system. The most important uses of the ultrasound medical system are the diagnoses of the heart (Figure 16A), and monitoring the baby and the expecting mother health during the pregnancy (Figure 16B). However, it is also used in diagnostics of kidneys, gallbladder, bladder, and other organs. There is a hand-held version of the entire image processing unit, which is not much larger than a smart phone (Figure 16C).

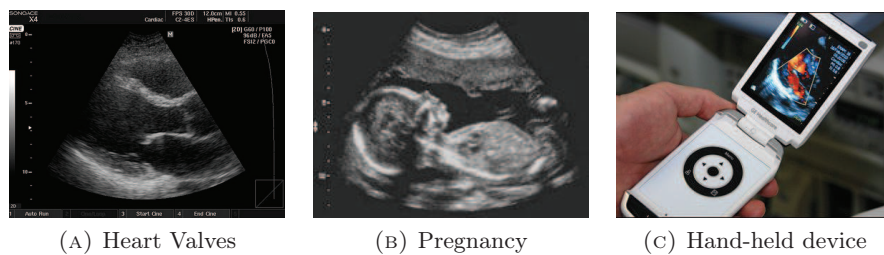| (A) Heart Valves | (B) Pregnancy | (C) Hand-held device |

FIGURE 16. Medical ultrasonography

The *radar imaging system* works similarly like the bat's ultrasound system, however, the radar can scan (sweep) a larger area (a spatial angle), and thus construct image of a very large scene. The invention of device called *laser*, which produces a narrow beam of light or infrared light pulses, enabled the construction of an imaging system called *ladar*. Ladar uses the short-time Fourier transform, also called the time-frequency transform, to process the reflected pulses into the range-Doppler image, which is a 2D phase space, where the horizontal coordinate is the radial distance and the vertical coordinate is the radial velocity of the illuminated object. The standard optical image of a satellite is in Figure 17A, and the corresponding range-Doppler image is in Figure 17B.

The compound eyes of insects are faster than eyes with lens and retina. U.S. Navy is studying fly compound eyes to help develop guidance systems for weapons, and more compact optical sensors (Figure 18). Hopefully, the intelligent vision processing will provide new technology for robotics, and other industrial or military applications. The cameras with compound lenses (Figure 18A) imitate the compound eyes of insects. The 100 megapixel camera (Figure 18B) is capable of producing images with $10,240 \times 10,240$ pixels.
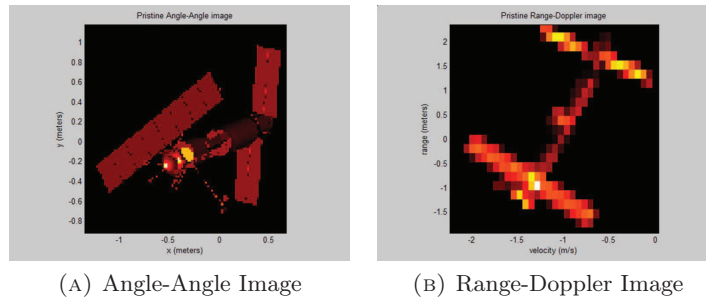
(A) Angle-Angle Image   (B) Range-Doppler Image

FIGURE 17. Images of a Satellite

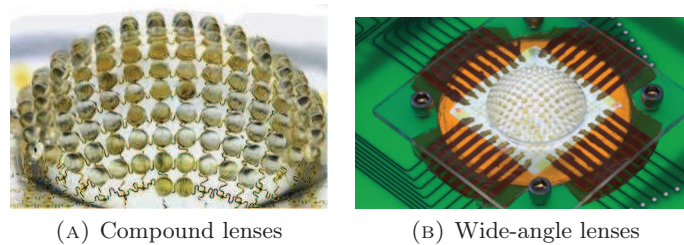(A) Compound lenses   (B) Wide-angle lenses

FIGURE 18. Cameras inspired by compound eyes of bugs

Stereo vision allows for depth perception and estimation of distances. However, the laser illumination of target provides an active and more precise estimation of the distance. The computer vision algorithms take a lot from eyes and compound eyes for processing the digital images. Panoramic images are created by stitching together shifted images to cover a larger view angle. The integration of images from multiple cameras is used in airplane cockpits. The planned Amazon's smartphone should have the total of 6 cameras, where 4 cameras will sense the hand gestures without the user touching the screen. Robots with vision sensors can perform complex functions such as a product assembly or reaching hard places. Hemispherical vision systems had been developed for monitoring and management of airports/airfields. This technology can be replaced by the multiple cameras modelling the compound eye of a honey bee. Multi-spectral vision systems put into one display the patches of images and pseudocolour images from different wavelengths, for example, from infrared or other invisible wavelengths.

Hyperspectral vision systems split the visible spectrum into disjoint intervals, say, 100 intervals, and calculate the pixel intensities for each interval.

The resulting image is actually a stack of 100 images, or one 3D image, called datacube (Figure 19A). Each bandwidth, corresponding to an interval of the spectrum, has a specific sensitivity to the chemical structures of regions in the field of view. Note that there are two layers in the datacube without data. These are caused by atmospheric absorption bands, specifically, H_2O at 1.5 and 2.0 microns. Many useful properties can be detected when looking at forests, fields, marshes, valleys, etc. (Figure 19B).

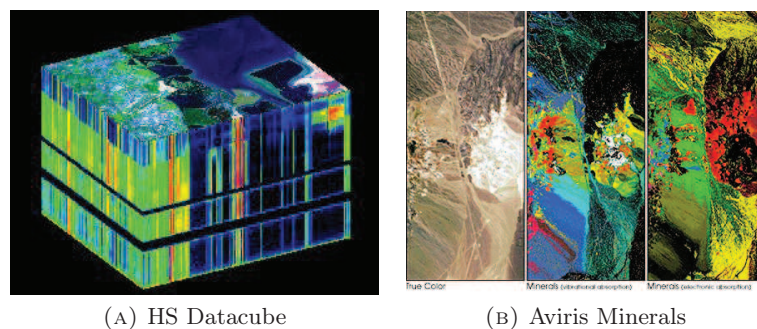(A) HS Datacube   (B) Aviris Minerals

FIGURE 19. Remote Sensing Images

The mantis shrimp (Wikipedia: Mantis shrimp) has one of the most elaborate visual systems ever discovered in animal kingdom. Each of two eyes carry 16 photoreceptor pigments which allow to perceive both polarized light and multispectral images. Moreover, each eye possesses trinocular vision and depth perception.

4. Camouflage

The *camouflage* used by insects, sea creatures and other species appears to protect them from predators, and shows that their evolution and brain are surprisingly advanced. The camouflage consists in mimicking the background, and this function is either permanent or adaptive. A moth or a beetle have very similar patterns to tree barks, where they rest (Figure 20).
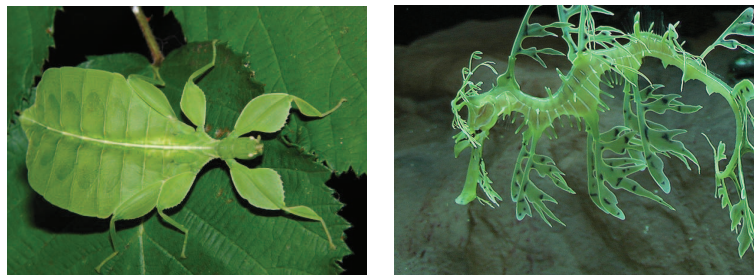
(A) Pine Hawk Moth   (B) Monochamus Notatus

FIGURE 20. Moth and beetle camouflaged on tree bark

The Leaf Insect resembles leaves of plants in its environment (Figure 21A). Similarly, the Leafy Sea Dragon has shapes almost identical to leaves in its sea environment (Figure 21B).

(A) Leaf insect   (B) Leafy sea dragon

FIGURE 21. Insect and sea dragon look like leaves

The adaptive camouflage depends on living environment. A fish can create a cover-up by mimicking the color patterns of an octopus (Figure 22).

FIGURE 22. Fish and octopus

The brain of this fish performs very complex image processing algorithms to change colors of its skin according to what it saw before assuming the camouflaged position.

This fish behaviour inspires some researchers to work on invisibility devices. First is the kind of cloth, which becomes invisible, when the surface, turned to the observer, maps the background behind the person wearing the cloth. To the observer it looks like there is nobody there because the seen background is not interrupted by the spot occupied by the person. Except the invisibility of objects in visible light, there are also studied surfaces and materials making objects invisible to radars, such as stealth planes, stealth helicopters and stealth drones.

REFERENCES

[1] John A. Adam, *Mathematics in Nature: Modelling patterns in the natural world*, Princeton University Press 2006.
[2] John D. Barrow, *The Artful Universe*, Clarendon Press, Oxford 1995.
[3] Richard Dawkins, *Climbing Mount Improbable*, W. W. Norton and Company, New York London 1996.
[4] Harry Henderson, *Mathematics: Powerful Patterns into Nature and Society*, Chelsea House 2007.
[5] Martin A Nowak, *Evolutionary Dynamics; Exploring the Equations of Life*, The Belknap Press of Harvard University Press 2006.
[6] Max Tegmark, *Our Mathematical Universe: My Quest for the Ultimate Nature of Reality*, Knopf, New York 2014.
[7] Harun Yahya, *Miracle in the eye*, 3rd edition, Global Publishing, Istanbul, Turkey, 1999.
[8] Harun Yahya, *Engineering in Nature*, Global Publishing, Istanbul, Turkey, 2007.

Figure Credits

Figure 1A: 169.237.77.3/news/valleycarpenterbees.html

Figure 1B: webecoist.momtastic.com/2010/04/25/feats-of-strength-6-amazingsurprising-animal-superpowers/

Figure 2A: thomasshahan.com/#photos

Figure 2B: commons.wikimedia.org/wiki/File:Anterior_Median_and_Anterior_Lateral_Eyes_of_a_Phidippus_princeps_Jumping_Spider.jpg

Figure 3A: impressivemagazine.com/2012/06/11/striking-colors-starfish/

Figure 3B: en.wikipedia.org/wiki/Crown-of-thorns_starfish

Figure 4A: www.phy.duke.edu/~hsg/162/images/scallop-eye.html

Figure 4B: www.wildlife-art.net/kimberley131.html

Figure 5A: forshorefishing.blogs.theledger.com/16792/beauty-is-in- the-eye-of-the-one-holding-the-bay-scallop/

Figure 5B: www.phy.duke.edu/~hsg/162/images/scallop-eye.html

Figure 6A: commons.wikimedia.org/wiki/File:Eyes_of_a_Holcocephala_fusca_Robber_Fly.jpg

Figure 6B: pixelcurse.com/photography/50-amazing-images-from-the-eye-of-macro-lens; www.flickr.com/photos/rundstedt/4054542431/

Figure 7: en.wikipedia.org/wiki/Stoplight_loosejaw

Figure 8: www.duiops.net/seresvivos/galeria_plantas_14.html; www.duiops.net/seresvivos/galeria/plantasflores/Leaf%20with%20water.jpg

Figure 9A: micro.magnet.fsu.edu/primer/anatomy/introduction.html www.olympusmicro.com/primer/anatomy/introduction.html

Figure 9B: www.olympusmicro.com/primer/anatomy/objectives.html

Figure 10: www.finnpathologists.co.uk/about.html; www.biomedcode.com/gr/en/content/preclinical-efficacy-platforms; www.shutterstock.com/pic-109591796/stock-photo-modern-microscope-equipped-with-digital-camera-computer-and-monitor.html?src=0-BWEn5fgM468CdffcY7Yg-1-20

Figure 11: www.theguardian.com/science/blog/2009/aug/25/galileo-telescope-anniversary

Figure 12: astro.berkeley.edu/~jrg/chabot/Slide11.JPG

Figure 13: spinoff.nasa.gov/spinoff2003/hm_9.html

Figure 14A: culturepology.wordpress.com/category/the-universe/; www.graymanwrites.com/blog/two-space-anniversaries-first-china-and-hubble-reaches-orbit/

Figure 14B: www.amazon.com/Meade-ETX70AT-Telescope-Tripod-Software/dp/B00005ATSR

Figure 15A: animals.nationalgeographic.com/animals/mammals/common-vampire-bat/

Figure 15B: insectbio.blogs.rice.edu/; charles-harvey.co.uk/batting-away-pesticide-use/

Figure 16A: www.medison.ru/uzi/eho405.htm

Figure 16B: community.babycenter.com/post/a36383725/level_2_ultrasound

Figure 16C: www.legitreviews.com/a-hand-held-ultrasound-scanner-fit-for-dr-mccoy_7429

Figure 17A: Matlab data archive of the author

Figure 17B: Created in Matlab by the author

Figure 18A: nanohub.org/resources/19612/watch?resid=19613; pmanewsline.com/2013/05/02/camera-captures-bugs-eye-like-image/#.VBnQPfldWqg

Figure 18B: ibnlive.in.com/news/chinese-institute-develops-100megapixel-camera/405546-11.html; engineering.illinois.edu/news/article/2013-04-29-bugs-view-inspires-new-digital-cameras-unique-imaging-capabilities

Figure 19A: aviris.jpl.nasa.gov/data/image_cube.html; www.sarracenia.com/astronomy/remotesensing/primer0350.html

Figure 19B: ces.iisc.ernet.in/hpg/envis/Remote/introfile27.htm

Figure 20A: insects.about.com/od/photography/ig/Sphinx-Moths/Pine-Hawk-Moth.htm

Figure 20B: bugguide.net/node/view/814232/bgimage

Figure 21A: planetapex.blogspot.com/2013/05/leaf-insect-amazing-walking-leaf.html; www.animalsinhands.com/zoo-animals/; litsciox.wordpress.com/2014/04/22/exhibition-and-public-talk-on-natural-mimicry-at-oxford-natural-history-museum-april-may-2014/

Figure 21B: flickriver.com/photos/tags/taxonomy%3Aorder%3Dsyngnathidae/interesting/

Figure 22: cestovanie.aktuality.sk/clanok/5773/novy-objav-podmorskeho-zivota-ryba-maskujuca-sa-na-chobotnicu/

Rastislav Telgársky
Department of Mathematics
Central New Mexico Community College
525 Buena Vista Drive, Albuquerque, NM 87106, U.S.A.
E-mail address: rtelgarsky@cnm.edu; rastislav@telgarsky.com