Scientific Issues
Jan Długosz University
in Częstochowa
Mathematics XX (2015)
23–28
DOI http://dx.doi.org/10.16926/m.2015.20.03

TIME CALCULATIONS IN SCHOOL

MARCIN ZIÓŁKOWSKI, LIDIA STĘPIEŃ, MARCIN RYSZARD STĘPIEŃ

Abstract

In this article we want to present some simple method of doing time calculations which is not often used by teachers and show that the discussed way does not have to be difficult for students and is based on strict mathematical rules.

Keywords: *theory of divisibility, equivalence class, time calculations*

1. Introduction

Every day we face the problem of time calculations. We often calculate when our train arrives at the terminal station or we want to know how much time we will spend on the plane during the flight to the foreign country. The life seems to be faster now, so we plan all our activities precisely, calculating time very scrupulously. Simple time calculations appear mainly at the first and second educational level and they are a big problem for both students and teachers. At the first and the second educational level the current core curriculum in mathematics [5] consists of contents connected with simple time calculations. Such tasks can be very difficult for students because time units are not based on decimal system so they cannot use arithmetic algorithms and have to use completion methods instead as it is shown in the following examples. Many books dedicated for students of second educational level contain the presented technique [3-4].

Example 1. *The train from Warsaw to Siedlce leaves at* 12.45. *What time does the train arrive at the terminal station if the travel lasts one hour and fifty-five minutes?*

The solution in mathematical books for students is the following one:

$$12.45\ (+1\ h) \to 13.45\ (+15\ min) \to 14.00\ (+40\ min) \to 14.40.$$

Marcin Ziółkowski — Jan Długosz University in Częstochowa.
Lidia Stępień — Jan Długosz University in Częstochowa.
Marcin Ryszard Stępień — Kielce University of Technology.

So the train arrives at Siedlce at 14.40.

Example 2. *The bus leaves Zakopane at* 21.45 *and arrives at Warsaw at* 9.10. *How long does the journey take?*

The solution:

$$21.45\ (+2\ h\ 15\ min) \to 24.00\ (+9\ h\ 10\ min) \to 9.10.$$

Then we have $2\ h\ 15\ min + 9\ h\ 10\ min = 11\ h\ 25\ min$. So the journey takes 11 hours and 25 minutes.

The above solutions seem to be very simple for students of early educational levels but the notation is longer than it has to be. On the other hand the solutions can create the problems connected with completion mechanism of time calculations. It is obvious that in time calculations we use addition and subtraction. The completion mechanism suggests that we use only addition. The second problem is that the above notation is not mathematically correct. Of course, we can simply use addition and subtraction to compute some exact time or time difference but it leads to the problems of exceeding real time intervals. It is the main reason why some of the teachers do not want to teach time calculations this way. In the next sections we show the method which is more intuitive and does not seem to be more difficult for students. We also present the mathematical point of view connected with the equivalence class concept.

2. Mathematical background

Let us consider the relation $\rho \subset \mathbb{N}^2 \times \mathbb{N}^2$ given by the formula:

$$(h_1, m_1)\rho(h_2, m_2) \Leftrightarrow \exists_{k,l\in\mathbb{Z}}\,(m_1 - m_2 = l \cdot 60 \wedge h_1 - h_2 + l = k \cdot 24) \quad (1)$$

It is easy to show that the relation (1) is reflexive, symmetric and transitive so it divides the set $\mathbb{N}^2$ into equivalence classes [1].

Proof.
If we take $k = l = 0$, we notice that for every (h, m) we have $(h, m)\rho(h, m)$, which proves reflexivity. If we assume that $(h_1, m_1)\rho(h_2, m_2)$, then it is simple to show that we also have $(h_2, m_2)\rho(h_1, m_1)$ as $m_2 - m_1 = -(m_1 - m_2)$ and $h_2 - h_1 + l = -(h_1 - h_2 - l)$ so the relation is symmetric. The right side of (1) can be rewritten as follows:

$$h_1 - h_2 + \frac{m_1 - m_2}{60} = 24 \cdot k. \quad (2)$$

We use this condition to prove that the relation is transitive. If we assume that $(h_1, m_1)\rho(h_2, m_2)$ and $(h_2, m_2)\rho(h_3, m_3)$, then we obtain

$$h_1 - h_2 + \frac{m_1 - m_2}{60} = 24 \cdot k_1$$

$$h_2 - h_3 + \frac{m_2 - m_3}{60} = 24 \cdot k_2$$

Thus

$$h_1 - h_3 + \frac{m_1 - m_3}{60} = 24 \cdot (k_1 + k_2). \qquad \square$$

We notice that the relation (1) lets introduce the equivalence between time notations. For example notations 1.70, 0.130, 25.70 and 2.10 can be understood as the different notations for the same time. It is also obvious that for every equivalence class there exists a representative that does not exceed the interval of real hours and minutes, i.e., $h \in [0, 23]$ and $m \in [0, 59]$, respectively.

3. Time calculations

In this section we introduce two operations connected with addition and subtraction that let solve time calculation tasks in easier way. These time operations could be mathematically defined as follows:

$$(h_1, m_1) + (h_2, m_2) = ((h_1 + h_2 + (m_1 + m_2) \div 60) \,\%\, 24, (m_1 + m_2) \,\%\, 60), \quad (3)$$

$$(h_1, m_1) - (h_2, m_2) = ((h_1 - h_2 + (m_1 - m_2) \div 60) \,\%\, 24, (m_1 - m_2) \,\%\, 60). \quad (4)$$

Operations $\div$ and $\%$, the integer division and the modulo operation, respectively, are the basic operations in the theory of divisibility (see [2] pp. 173). For example:

$$7 \div 3 = 2 \text{ and } 7 \,\%\, 3 = 1 \text{ as } 7 = 2 \cdot 3 + 1$$

$$-13 \div 5 = -3 \text{ and } -13 \,\%\, 5 = 2 \text{ as } -13 = -3 \cdot 5 + 2$$

The results of the operations (3) and (4) belong to the set $\mathbb{N}^2$ but also do not exceed real time intervals. However, they are difficult for students because of two division operations. We want to change calculations in such a way that it would suffice to use only addition or subtraction operations. It is possible because in life calculations the arguments do not exceed time intervals so the division operations are not necessary. We do calculations similarly to vector operations but we have to choose, during the calculation, the proper representative of equivalence class that replaces the argument. For example, in subtraction, if $m_1 < m_2$, then we replace (h_1, m_1) by $(h_1 - 1, m_1 + 60)$ and if $h_1 < h_2$ then we replace (h_1, m_1) by $(h_1 + 24, m_1)$. In addition, there is no problem with operations. Finally, we write the results also choosing the proper representative. If $m > 59$, then we replace (h, m)

by $(h+1, m-60)$. If $h > 23$, we only have to replace (h, m) by $(h-24, m)$. It is shown in the following examples:

$$3.40 + 4.50 = 7.90 = 8.30,$$

$$23.15 + 4.50 = 27.65 = 3.65 = 4.05,$$

$$12.10 - 3.30 = 11.70 - 3.30 = 8.40,$$

$$4.20 - 9.50 = 28.20 - 9.50 = 27.80 - 9.50 = 18.30.$$

This mechanism is very similar to the rational number calculations:

$$3\frac{5}{7} + 2\frac{4}{7} = 5\frac{9}{7} = 6\frac{2}{7},$$
$$7\frac{1}{6} - 2\frac{5}{6} = 6\frac{7}{6} - 2\frac{5}{6} = 4\frac{2}{6}.$$

So, it is educationally advisable to take the opportunity to teach these two skills at the similar time.

We also want to pay attention to the other problem. From the educational point of view we should not use time notation that involves dots as we do in this article. It may cause the problems connected with calculations in the decimal system - especially the use of arithmetic algorithms can lead to false results. So we should use the index notation which suggests the students that they deal with a different mathematical construction.

Example 3. *Mark went to the cinema on Saturday. The film started at* 14.50 *and lasted two hours and fifty minutes. When did it finish?*

Solution 1 (false result):

$14.50 + 2.50 = 17.00$ (an arithmetic algorithm as $1450 + 250 = 1700$),

Solution 2 (correct):

$14^{\underline{50}} + 2^{\underline{50}} = 16^{\underline{100}} = 17^{\underline{40}}$.

Example 4. *The concert ended at* 17.05 *and lasted three hours and fifty minutes. When did it start?*

Solution 1 (false result):

$17.05 - 3.50 = 13.55$ (an arithmetic algorithm as $1705 - 350 = 1355$),

Solution 2 (correct):

$17^{\underline{05}} - 3^{\underline{50}} = 16^{\underline{65}} - 3^{\underline{50}} = 13^{\underline{15}}$.

4. Final remarks

In this article we offer an alternative method of performing time calculations which involves only addition and subtraction. It is obvious that students have no problems with such operations at the first and the second educational level, so this method is a possible alternative to the completion methods. In addition, the completion methods have not good mathematical notation – so it is another reason to use the presented technique, especially if we work with talented students who take part in mathematical competitions. Of course, students have to be properly prepared for such calculations. But our technique seems to be more intuitive in some cases (calculating a time difference we use a subtraction - not addition) and it is based on some strict theoretical rules (hence the notation is mathematical). Besides, we use similar techniques in other areas of mathematics (for example when dealing with rational numbers), so it is educationally reasonable to take this calculation algorithm into account. The introduced technique can be also used as an interesting and practical task for university students who study equivalence class examples.

References

[1] H. Rasiowa. *Wstęp do matematyki współczesnej.* PWN. Warszawa, 2013.

[2] A. Mostowski, M. Stark. *Elementy algebry wyższej.* PWN. Warszawa, 1975.

[3] B. Dubiecka-Kruk, P. Piskorski, A. Gleirscher, E. Malicka, E. Pytlak. *Matematyka z pomysłem. Podręcznik do klasy czwartej szkoły podstawowej - część 1.* WSiP. Warszawa, 2015.

[4] L. Klama, E. Szelecka, A. Mazera, M. Pyziak, R.Miłek, R. Warzocha. *Matematyka z klasą. Podręcznik do klasy czwartej szkoły podstawowej.* LektorKlett. Poznań, 2015.

[5] `http://men.gov.pl/wp-content/uploads/2014/08/zalacznik_2.pdf` (2015, October 10^{th})

Received: October 2015

Marcin Ziółkowski
Jan Długosz University in Częstochowa,
Institute of Mathematics and Computer Science,
Al. Armii Krajowej 13/15, 42-200 Częstochowa, Poland
E-mail address: m.ziolkowski@ajd.czest.pl

Lidia Stępień
Jan Długosz University in Częstochowa,
Institute of Mathematics and Computer Science,
Al. Armii Krajowej 13/15, 42-200 Częstochowa, Poland
E-mail address: l.stepien@ajd.czest.pl

Marcin Ryszard Stępień
Kielce University of Technology,
Chair of Mathematics,
Al. Tysiąclecia Państwa Polskiego 7, 25-314 Kielce, Poland
E-mail address: mstepien@tu.kielce.pl