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HIGH-SPEED MODULAR STRUCTURES FOR PARALLEL COMPUTING IN THE SPACE OF ORTHOGONAL PROJECTIONS

Summary

The numerical-analytical method of digital signal processing based on the modular model of space of orthogonal projections are presented in the article. This gives new possibilities for the high-performance processing of discrete signals on numerical-analytical level at the realization not only arithmetic but also more complicated operations such as convolution, correlation, algorithms of spectral analysis and others.

Keywords: Digital signal processing, space of orthogonal projections, Chebyshev polynomials, modular computing structures, modular arithmetic

The decomposition of one-dimensional and many-dimensional signals by systems of orthogonal functions provide highly convenient and effective basis for still developing flexible numerical-analytical technique oriented on creation of methods and algorithms of high-performance adaptive digital signal processing (DSP) [1, 2].

Application of numerically-analytical technique to DSP systems is based on approximation of digital signals $y(u) = \{y_0, y_1, \dots, y_{n-1}\}$ by truncated orthogonal series [2]:

$$y(u) \approx \tilde{y}(u) = \sum_{l=0}^{n-1} a_l T_l(u), \qquad (1)$$

where $T_l(u) = cos\big(l\, arccos(u)\big)$ $(u \in [-1; 1]; l = 0,1, \dots)$ are the Chebyshev polynomials of the first kind, the expansion coefficients $a_0, a_1, \dots, a_{n-1}$ are computed by calculation relation

$$a_l = \sum_{r=0}^{N-1} h_{l,r} y_r \quad (l = 0,1,\dots,n-1), \tag{2}$$

at that

$$h_{l,r} = \frac{\sqrt{(S(-l)+l)\pi}}{N} T_l(u_r) \quad (l = 0,1,\dots,n-1, \;\; r = 0,1,\dots,N-1)$$

the samples $h_{l,r}$ are calculated at the nodes

$$u_r = cos\left(\frac{2r+1}{2N}\pi\right) \quad (r = 0,1,\dots,N-1), \tag{3}$$

$S(x)$ is a function associated with the sign

$$S(x) = \begin{cases} 0 \; if \; x \geq 0, \\ 1 \; if \; x < 0. \end{cases}$$

On the one hand, such approach allows us to realize effective information compression (N/n times more) obtained at the expense of conversion from the explicit digital form $\{y_0, y_1, \dots, y_{N-1}\}$ of a signal $y(u)$ to analytic form $\{a_0, a_1, \dots, a_{n-1}\}$; on the other hand, it allows us to increase a computation speed.

High capability for increase of DSP system performance in the context of numerical-analytical modular technique [2] is objectively conditioned by two main factors:

1) reduction of computing complexity of DSP algorithms for vectors $\langle a_0, a_1, \dots, a_{n-1} \rangle$ in the space of orthogonal projections (SOP) in comparison with analogous processing algorithms of the initial digital signals $\{y_0, y_1, \dots, y_{N-1}\}$,

2) the natural parallelism of minimal redundant modular computing structures applying both at computer-arithmetic and algorithmic levels [3-5].

Implementation of specified possibilities first of all demands obtaining of the base analytical relations and invariants in the SOP and also obtaining of theirs modular versions. The present article first of all is devoted to this problem exactly.

First let us consider the analytical methods of performance of signals addition, subtraction and multiplication in the SOP.

As regards the additive procedures, the calculation relations for them imply directly from (1). Let two arbitrary vectors $y^{(1)} = \langle a_0^{(1)}, a_1^{(1)}, \dots, a_{n-1}^{(1)} \rangle$ and $y^{(2)} = \langle a_0^{(2)}, a_1^{(2)}, \dots, a_{n-1}^{(2)} \rangle$ be given. The following signals

$$y^{(1)}(u) = \sum_{l=0}^{n-1} a_l^{(1)} T_l(u) \tag{4}$$

and

$$y^{(2)}(u) = \sum_{l=0}^{n-1} a_l^{(2)} T_l(u) \qquad (5)$$

correspond to them accordingly.

Since from (4) and (5) it follows that

$$y^{(1)}(u) \pm y^{(2)}(u) = \sum_{l=0}^{n-1} \left(a_l^{(1)} \pm a_l^{(2)}\right) T_l(u),$$

then for the sum and difference of vectors $y^{(1)}$ and $y^{(2)}$ the following formula is true

$$y^{(1)} \pm y^{(2)} = \langle a_0^{(1)} \pm a_0^{(2)}, a_1^{(1)} \pm a_1^{(2)}, \dots, a_{n-1}^{(1)} \pm a_{n-1}^{(2)} \rangle. \qquad (6)$$

For modular model of SOP the rule (6) becomes [2, 3]

$$y^{(1)} \pm y^{(2)} = \langle \left(\left|\alpha_1^{0,1} \pm \alpha_1^{0,2}\right|_{m_1}, \left|\alpha_2^{0,1} \pm \alpha_2^{0,2}\right|_{m_2}, \dots, \left|\alpha_k^{0,1} \pm \alpha_k^{0,2}\right|_{m_k} \right);$$

$$\left(\left|\alpha_1^{1,1} \pm \alpha_1^{1,2}\right|_{m_1}, \left|\alpha_2^{1,1} \pm \alpha_2^{1,2}\right|_{m_2}, \dots, \left|\alpha_k^{1,1} \pm \alpha_k^{1,2}\right|_{m_k} \right);$$

$$\left(\left|\alpha_1^{n-1,1} \pm \alpha_1^{n-1,2}\right|_{m_1}, \left|\alpha_2^{n-1,1} \pm \alpha_2^{n-1,2}\right|_{m_2}, \dots, \left|\alpha_k^{n-1,1} \pm \alpha_k^{n-1,2}\right|_{m_k} \right) \rangle, \qquad (7)$$

where $\left(\alpha_1^{l,t}, \alpha_2^{l,t}, \dots, \alpha_k^{l,t}\right)$ is a minimal redundant modular code (MRMC) of the numerator of fraction $A_l^{(t)}/S$ approximating coefficient $a_l^{(t)}$ $(l = 0,1, \dots, n - l; t = 1,2)$ (see (2)).

Derivation of a calculation relation for multiplication of signals can also be realized by means of the considered method, i.e. by conversion of product of expressions (4) and (5) to the truncated orthogonal series of type (1).

Taking into account an equality

$$T_p(u).T_q(u) = cos(.p.arccos(u).)\ cos(.q.arccos(u).) =$$

$$= \frac{1}{2}\left(cos\left((p+q)arccos(u)\right) + cos\left((p-q)arccos(u)\right)\right) = \frac{1}{2}\left(T_{p+q}(u) + T_{|p-q|}(u)\right)$$

in accordance with (4) and (5) we have

$$y^{(1)} \times y^{(2)} = \left(\sum_{p=0}^{n-1} a_p^{(1)} T_p(u)\right)\left(\sum_{q=0}^{n-1} a_p^{(2)} T_q(u)\right) =$$

$$\sum_{p=0}^{n-1}\sum_{q=0}^{n-1} a_p^{(1)} a_q^{(2)} T_p(u) T_q(u) = \frac{1}{2}\sum_{p=0}^{n-1}\sum_{q=0}^{n-1} a_p^{(1)} a_q^{(2)} \left(T_{p+q}(u) + T_{|p-q|}(u)\right) =$$

$$\frac{1}{2}\sum_{l=0}^{n-1}\left(\sum_{p=0}^{l} a_p^{(1)} a_{l-p}^{(2)} + S(-l)\sum_{p=0}^{n-l-1} a_p^{(1)} a_{l-p}^{(2)} + \sum_{p=l}^{n-1} a_p^{(1)} a_{l-p}^{(2)}\right) T_l(u) +$$

$$\frac{1}{2}\sum_{l=n}^{2n-2}\left(\sum_{p=l-n+1}^{n-1} a_p^{(1)} a_{l-p}^{(2)}\right)T_l(u). \quad (8)$$

Cutting the items containing polynomials of power higher than $n-1$, i.e. polynomial segment

$$\Delta_n(u) = \frac{1}{2}\sum_{l=n}^{2n-2}\left(\sum_{p=l-n+1}^{n-1} a_p^{(1)} a_{l-p}^{(2)}\right) T_l(u), \quad (9)$$

and replacing coefficients $a_s^{(t)}$ by their rational approximations $A_s^{(t)}/S$ $(s = 0, 1, \dots, n-1, t = 1, 2)$, for the numerators A_l of coefficients $a_l \approx A_l/S (l = 0, 1, \dots, n-1)$ of the Chebyshev form $\sum_{l=0}^{n-1} a_l T_l(u)$ for approximation of the product of signals $y^{(1)}$and $y^{(2)}(u)$ from (8) we obtain the following expression:

$$A_l = \left]\left(\sum_{p=0}^{l} A_p^{(1)} A_{l-p}^{(2)} + S(-l)\sum_{p=0}^{n-l-1} A_p^{(1)} A_{p+l}^{(2)} + \sum_{p=l}^{n-1} A_p^{(1)} A_{p-l}^{(2)}\right)/(2S)\right[, \quad (10)$$

where $]x[$denotes the rounding operation for number x.

An error $\Delta_n(u)$ of approximation $y^{(1)}(u) \times y^{(2)}(u)$ by the n-terms Chebyshev form for every $u \in [-1;\ 1]$ is a quantity of the same order as the first omitted term, i.e.

$$\sup_{u\in[-1;1]} \{\Delta_n(u)\} \approx a_n$$

Therefore, according to (9)

$$|\Delta_n(u)| \le \frac{1}{2}\sum_{p=l}^{n-1} a_p^{(1)} a_{n-p}^{(2)}.$$

Implementation of the calculation relations (10) by means of minimal redundant modular number system (MRMNS) can be realized by using two methods: with the aid of pseudo-modular or modular version of minimal redundant modular arithmetic (MRMA) [5].

In the first method all multiplications of the coefficients $A_p^{(1)}$ and $A_q^{(2)}$ $(p, q \in Z)$ are performed with rounding by scaling of the products on a scale 2S (see (10)).

The organization of calculations in accordance with (10) in the context of the modular version of MRMA requires application of the range **D** which includes every possible integer values

$$A(l) = \sum_{p=0}^{l} A_p^{(1)} A_{l-p}^{(2)} + S(-l)\sum_{p=0}^{n-l-1} A_p^{(1)} A_{p+l}^{(2)} + \sum_{p=1}^{n-1} A_p^{(1)} A_{p-1}^{(2)}$$

$$(l = 0, 1, \dots, n-1). \quad (11)$$

At the prescribed value l, the total number of items in (11) equals to $N(l) = n + (n - l)\, S(-l) + 1$, and it is easy to see that $\max_l\{N(l)\} = N(l) = 2n$. Hence, the estimation $|A(l)| \leq max\{|A(1)|\} \leq 2n\, A_{max}^2$, where $A_{max} = max\left\{\left|A_l^{(t)}\right|\right\}$, occurs. According to (2) and [2], $A_{max} = P^2$. Therefore, the parameter M of range $D = \{-M, -M+1, \dots, M-1\}$ of MRMNS allowing realization of formal calculations according to (11) as well as to (10) for all $l = 0,1, \dots, n-1$ should satisfy the inequality

$$M > 2nP^4. \tag{12}$$

If for example $n = 16$, $P = 2^{11}$, then according to (12) the power of the dynamic range D should be chosen from a condition $|\boldsymbol{D}| = 2M > 2^{50}$. This demands the use of 11 modules of 3-5 bits width.In contrast of additive operations with vectors

$$y^{(1)} = \langle a_0^{(1)}, a_1^{(1)}, \dots, a_{n-1}^{(1)} \rangle$$

and

$$y^{(2)} = \langle a_0^{(2)}, a_1^{(2)}, \dots, a_{n-1}^{(2)} \rangle$$

(see (6), (7)), the procedure of multiplication of these vectors (10) includes not only modular operations which are realized during the process of calculation of the MRMC of numbers (11)

$$\Big(\Big|\textstyle\sum_{p=0}^{l}\big|\alpha_1^{p,1}\alpha_1^{l-p,2}\big|_{m1} + S(-l)\sum_{p=0}^{n-l-1}\big|\alpha_1^{p,1}\alpha_1^{l+p,2}\big|_{m1} + \sum_{p=l}^{n-1}\big|\alpha_1^{p,1}\alpha_1^{p-l,2}\big|_{m1}\Big|_{m1},$$

$$\Big|\textstyle\sum_{p=0}^{l}\big|\alpha_2^{p,1}\alpha_2^{l-p,2}\big|_{m2} + S(-l)\sum_{p=0}^{n-l-1}\big|\alpha_2^{p,1}\alpha_2^{p+l,2}\big|_{m2} + \sum_{p=l}^{n-1}\big|\alpha_2^{p,1}\alpha_2^{p-l,2}\big|_{m2}\Big|_{m2}$$

$$\Big|\textstyle\sum_{p=0}^{l}\big|\alpha_k^{p,1}\alpha_k^{l-p,2}\big|_{m_k} + S(-l)\sum_{p=0}^{n-l-1}\big|\alpha_k^{p,1}\alpha_k^{p+l,2}\big|_{m_k} + \sum_{p=l}^{n-1}\big|\alpha_k^{p,1}\alpha_k^{p-l,2}\big|_{m_k}\Big|_{m_k}\Big)$$

but also includes *n* nonmodular operations of scaling *A*(*l*) which are necessary for computation of required set of the MRMC of the rounded Chebyshev projections of product

$$y^{(1)} \times y^{(2)} = \langle\left(\left|\rceil\tfrac{A(0)}{2S}\lceil\right|_{m_1}, \left|\rceil\tfrac{A(0)}{2S}\lceil\right|_{m_2}, \dots, \left|\rceil\tfrac{A(0)}{2S}\lceil\right|_{m_k}\right);$$

$$\left(\left|\rceil\tfrac{A(l)}{2S}\lceil\right|_{m_1}, \left|\rceil\tfrac{A(l)}{2S}\lceil\right|_{m_2}, \dots, \left|\rceil\tfrac{A(l)}{2S}\lceil\right|_{m_k}\right); \dots;$$

$$\left(\left|\rceil \frac{A(n-l)}{2S} \lceil\right|_{m_1}, \left|\rceil \frac{A(n-1)}{2S} \lceil\right|_{m_2}, \dots, \left|\rceil \frac{A(n-1)}{2S} \lceil\right|_{m_k}\right)\rangle.$$

For practical applications the SOP with $n = 8$ are quite acceptable. For this case the integer values (11) are presented below:

$$A(0) = 2A_0^{(1)}A_0^{(2)} + A_1^{(1)}A_1^{(2)} + A_2^{(1)}A_2^{(2)} + \cdots + A_7^{(1)}A_7^{(2)};$$

$$A(1) = 2A_0^{(1)}A_1^{(2)} + A_1^{(1)}A_0^{(2)} + A_1^{(1)}A_2^{(2)} + A_2^{(1)}A_1^{(2)} +$$

$$A_2^{(1)}A_3^{(2)} + A_3^{(1)}A_2^{(2)} + \cdots + A_6^{(1)}A_7^{(2)} + A_7^{(1)}A_6^{(2)};$$

$$A(2) = 2\left(A_0^{(1)}A_2^{(2)} + A_2^{(1)}A_0^{(2)}\right) + A_1^{(1)}A_1^{(2)} + A_1^{(1)}A_3^{(2)} +$$

$$A_3^{(1)}A_1^{(2)} + A_2^{(1)}A_4^{(2)} + A_4^{(1)}A_2^{(2)} + \cdots + A_5^{(1)}A_7^{(2)} + A_7^{(1)}A_5^{(2)};$$

$$A(3) = 2\left(A_0^{(1)}A_3^{(2)} + A_3^{(1)}A_0^{(2)}\right) + A_1^{(1)}A_2^{(2)} + A_2^{(1)}A_1^{(2)} +$$

$$A_1^{(1)}A_4^{(2)} + A_4^{(1)}A_1^{(2)} + A_2^{(1)}A_5^{(2)} + A_5^{(1)}A_2^{(2)} + A_3^{(1)}A_6^{(2)} +$$

$$A_6^{(1)}A_3^{(2)} + A_4^{(1)}A_7^{(2)} + A_7^{(1)}A_4^{(2)};$$

$$A(4) = 2\left(A_0^{(1)}A_4^{(2)} + A_4^{(1)}A_0^{(2)}\right) + A_1^{(1)}A_3^{(2)} + A_2^{(1)}A_2^{(2)} +$$

$$A_3^{(1)}A_1^{(2)} + A_1^{(1)}A_5^{(2)} + A_5^{(1)}A_1^{(2)} + A_2^{(1)}A_6^{(2)} + A_6^{(1)}A_2^{(2)} +$$

$$A_3^{(1)}A_7^{(2)} + A_7^{(1)}A_3^{(2)};$$

$$A(5) = 2\left(A_0^{(1)}A_5^{(2)} + A_5^{(1)}A_0^{(2)}\right) + A_1^{(1)}A_4^{(2)} + A_2^{(1)}A_3^{(2)} + A_3^{(1)}A_2^{(2)} +$$

$$A_4^{(1)}A_1^{(2)} + A_1^{(1)}A_6^{(2)} + A_6^{(1)}A_1^{(2)} + A_2^{(1)}A_7^{(2)} + A_7^{(1)}A_2^{(2)};$$

$$A(6) = 2\left(A_0^{(1)}A_6^{(2)} + A_6^{(1)}A_0^{(2)}\right) + A_1^{(1)}A_5^{(2)} + A_2^{(1)}A_4^{(2)} +$$

$$A_3^{(1)}A_3^{(2)} + A_4^{(1)}A_2^{(2)} + A_5^{(1)}A_1^{(2)} + A_1^{(1)}A_7^{(2)} + A_7^{(1)}A_1^{(2)};$$

$$A(7) = 2\left(A_0^{(1)}A_7^{(2)} + A_7^{(1)}A_0^{(2)}\right) + A_1^{(1)}A_6^{(2)} + A_2^{(1)}A_5^{(2)} + \cdots +$$

$$A_5^{(1)}A_2^{(2)} + A_6^{(1)}A_1^{(2)}$$

The numerical-analytical DSP methods based on the use of modular model of SOP also allow us to realize more complex operations and procedures with discrete signals, in particular such as geometrical transformations, convolution, correlation, algorithms of spectral analysis, etc.

As an example consider execution of convolution procedure of the signals $y^{(1)}(u)$ and $y^{(2)}(u)$ (see (4), (5)). The problem consists in synthesis of an analytical form (1) for signal

$$y(u) = y^{(1)}(u) \otimes y^{(2)}(u) = \int_{-1}^{1} y^{(1)}(u-v) y^{(2)}(v) dv \quad (u \epsilon [-1; 1]).$$

It is supposed that $y^{(t)}(u)(t = 1, 2)$ and $T_l(u)$ $(l = 0, 1, \dots, n-1)$ are predetermined at points $u \pm 2$ by rules $y^{(t)}(u \pm 2) = y^{(t)}(u), T_l(u \pm 2) = T_l(u)$ for all $u \in [-1; 1]$.

Taking into account (4) and (5), we have

$$y(u) = \int_{-1}^{1} y^1(u-v)\, y^2(v) dv = \int_{-1}^{1} \left(\sum_{p=0}^{n-1} a_p^{(1)} T_p(u-v)\right) \left(\sum_{q=0}^{n-1} a_q^{(2)} T_q(v)\right) dv =$$

$$\sum_{p=0}^{n-1} \sum_{q=0}^{n-1} a_p^{(1)} a_q^{(2)} \int_{-1}^{1} T_p(u-v) T_q(v) dv = \sum_{p=0}^{n-1} \sum_{q=0}^{n-1} a_p^{(1)} a_q^{(2)} \left(T_p \otimes T_q\right). \quad (13)$$

Cyclic convolution $T_{p,q}(u) = T_p \otimes T_q$ of Chebyshev polynomials $T_p(u)$ and $T_q(u)$ can be previously computed for any value of the variable u, for example at the nodes (3) of interval [-1; 1]. Therefore, expression (13) is rather convenient and effective calculated relation for desired convolution y(u) in a case when it is received in the explicit discrete form. If the signal $T_{p,q}(u)$ in (13) is replaced by its analytical representation

$$T_{p,q}(u) = \sum_{l=0}^{n-1} a_{p,q,l} T_l(u),$$

where $a_{p,q,l}$ is the preliminarily computed coefficient (see (2)), then $y(u)$ is reduced to an analytical form

$$y(u) = y^{(1)}(u) \otimes y^{(2)}(u) = \sum_{l=0}^{n-1} \left(\sum_{p=0}^{n-1} a_p^{(1)} \sum_{q=0}^{n-1} a_q^{(2)}\, a_{p,q,l}\right) T_l(u). \, (14)$$

Let $A_{p,q,l}$ denote the numerator of rational function $A_{p,q,l}/S$ approximating a projection $a_{p,q,l}$. Then following (14), the calculation relation for coefficients of Chebyshev orthogonal approximation $y(u) = \sum_{l=0}^{n-1} a_l T_l\,(u)$ of convolution $y(u) = y^{(1)}(u) \otimes y^{(2)}(u)$can be represented in the form

$$a_l = \frac{A_l}{S}; A_l = \rceil \frac{A(l)}{S} \lceil; A(l) = \sum_{p=0}^{n-1} A_p^{(1)}\, A_p(l); A_p(l) = \rceil \frac{A(p,l)}{S} \lceil;$$

$$A(p,l) = \sum_{q=0}^{n-1} A_q^{(2)} A_{p,q,l} (l = 0,1,\dots,n-1). \tag{15}$$

Calculation of vector $y^{(1)} \otimes y^{(2)} = \langle a_0, a_1, \dots, a_{n-1} \rangle$ according to (15) in the modular version of MRMA consists of two consecutive steps. On the first step, in the formal calculation mode the MRMC

$$\left(|A(p,l)|_{m_1}, |A(p,l)|_{m_2}, .., |A(p,l)|_{m_k}\right) =$$

$$\left(\left|\sum_{q=0}^{n-1} \left|\alpha_1^{q,2} \alpha_1^{p,q,l}\right|_{m_1}\right|_{m_1}, \left|\sum_{q=0}^{n-1} \left|\alpha_2^{q,2} \alpha_2^{p,q,l}\right|_{m_2}\right|_{m_2}, \dots, \left|\sum_{q=0}^{n-1} \left|\alpha_k^{q,2} \alpha_k^{p,q,l}\right|_{m_k}\right|_{m_k}\right)$$

$$\left(\alpha_i^{p,q,l} = \left|A_{p,q,l}\right|_{m_i} (i = 1,2,\dots,k)\right) \tag{16}$$

are evaluated for all numbers $A(p,l)(p,l = 0,1,\dots,n-1)$ with following scaling by *S*. On the second step, with the help of MRMC

$$\left(\alpha_1^{(p)}(l), \alpha_2^{(p)}(l), .., \alpha_k^{(p)}(l)\right) =$$

$$\left(\left|A_p(l)\right|_{m_1}, \left|A_p(l)\right|_{m_2}, \dots, \left|A_p(l)\right|_{m_k}\right)$$

and $\left(\alpha_1^{p,1}, \alpha_2^{p,1}, \dots, \alpha_k^{p,1}\right)$ for values $A_p(l)$ and $A_p^{(1)}$ respectively the MRMC

$$\left(|A(l)|_{m_1}, |A(l)|_{m_2}, \dots, |A(l)|_{m_k}\right) = \left(\left|\sum_{p=0}^{n-1} \left|\alpha_1^{p,1} \alpha_1^{p}(l)\right|_{m_1}\right|_{m_1}\right.$$

$$\left.\left|\sum_{p=0}^{n-1} \left|\alpha_2^{p,1} \alpha_2^{p}(l)\right|_{m_2}\right|_{m_2}, \dots, \left|\sum_{p=0}^{n-1} \left|\alpha_k^{p,1} \alpha_k^{p}(l)\right|_{m_k}\right|_{m_k}\right) \tag{17}$$

of numbers $A(l)$ are defined, as well as their rounded values

$$A_l = \left(\alpha_1^{(l)}, \alpha_2^{(l)}, \dots, \alpha_k^{(l)}\right) \left(\alpha_i^{(l)} = |A_l|_{m_i} (i = 1,2,\dots,k)\right)$$

for all $l = 0, 1, \dots n-1$.

From the considerations stated above it is seen that in the context of developed numerical-analytical method calculation of cyclic convolution decomposes into $n^2 + n$ practically identical elementary procedures, each of them including the modular computing segment described by expression of a type (16) or (17) and scaling of result of modular calculations by *S*.

Thus, the developed method provides exclusively high extent of unification of base computer procedures. From the viewpoint of practical realization this is of great importance.

It is necessary to notice that because of commutative property of the cyclic convolution $T_p \otimes T_q$, expressed by the equality

$$\int_{-1}^{1} T_p\,(u-v)T_q(v)dv = \int_{-1}^{1} T_q\,(u-v)T_p(v)dv,$$

which involves coincidence of factors $a_{p,q,l}$ and $a_{q,p,l}$ for all $p, q = 0, 1, \dots n-1$, for realization of the offered numerical-analytical method of signal convolution, the simplified procedures considering symmetry of matrixes $[ap, q, l](l = 0,1, \dots, n–1)$ can be synthesized.

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BARDZO SZYBKIE STRUKTURY MODULARNE DLA OBLICZEŃ RÓWNOLEGŁYCH W PRZESTRZENI RZUTÓW ORTOGONALNYCH

Summary

W artykule przedstawiono liczbowo-analityczną metodę cyfrowego przetwarzania sygnałów, która oparta jest na wykorzystaniu modułowego modelu przestrzeni rzutów ortogonalnych**.** Daje ona nowe możliwości dla wysokowydajnego przetwarzania sygnałów dyskretnych na liczbowo-analitycznym poziomie przy realizacji nie tylko arytmetycznych, ale również bardziej skomplikowanych operacji takich, w szczególności, jak splot, korelacja, algorytmy widmowej analizy i inne.

Keywords: cyfrowe przetwarzanie obrazów, przestrzeń rzutów ortogonalnych, wielomiany Czebyszewa, modularne struktury obliczeniowe, arytmetyka modularna