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MODULAR PRINCIPLES OF HIGH-SPEED ADAPTIVE FILTRATION OF DISCRETE SIGNALS

Abstract

The high-speed method of adaptive filtration of discrete signals based on the minimal redundant modular coding is presented in the article. The distinctive feature of the offered implementation of FIR filters consists in application of a new tabular multiplication of positional numbers by the fixed constants with obtaining the products in the minimum redundant modular number system.

Keywords: digital signal processing, digital filters, modular computing structures, modular number system, modular arithmetic

As it is known, the discrete linear systems with constant parameters are of fundamental importance in the theory and applications of digital signal processing (DSP) [1–4]. The problem of creating high speed digital filters allowing effectively execution of high-intensity streams of adaptive filtration operations in real time is of great significance in numerous modern applications of DSP.

The digital filtration represents the field of application in which all the fundamental advantages of modular computing structures can be implemented [5–7]. This is due to the fact that procedures of digital filtration possess high modularity. Their performance is reduced to calculation of sequences of linear combinations of residues with respect to modules of modular number systems (MNS) and also to the scaling operations which summarized number is rather insignificant in percentage terms. This fact allows construction of modular digital filters exceeding positional analogues at the data processing speed essentially. In addition, the filters based on MNS are characterized by higher accuracy.

Let us consider in detail the features of implementation of high-speed digital filters in minimum redundant MNS. In the most general case the linear digital filter of the N th order (N is the natural number) is specified by a difference equation with constant coefficients of the form:

$$y(l) = \sum_{n=0}^N a_n x(l-n) - \sum_{n=1}^N b_n y(l-n), \dots (l = 0, 1, \dots) \quad (1)$$

and by the initial samples $x(-r), y(-r)$ ($r = 1, 2, \dots, N$) of the input $\{x(n)\}$ and output $\{y(l)\}$ signals

(a_n and b_n are the real or complex coefficients; $|a_n| + |b_n| \neq 0$) [8]

Approximating coefficients a_n and b_n , which hereinafter are supposed real, by the fraction A_n/Q and B_n/Q respectively, where $A_n = \lfloor Qa_n \rfloor$, $B_n = \lfloor Qb_n \rfloor$ (Q is the fixed natural scale defining accuracy of approximation, $\lfloor x \rfloor$ denotes the nearest integer number to real x), we will substitute the equation (1) by the following approximate model:

$$y(l) = \frac{1}{S(l)} \left(\sum_{n=0}^N A_n x(l-n) - \sum_{n=1}^N B_n y(l-n) \right) (l = 0, 1, \dots), \quad (2)$$

Where $S(l) = QQ(l)$; $Q(l)$ is some scale which with the scaling factor Q ensures that the output sample $y(l)$ is in the required range; all the samples $x(n)$ ($n \geq -N$) of input signal and initial samples $y(-r)$ ($r = 1, 2, \dots, N$) of output signal are supposed to be integer.

The linear recursive digital filters of the N th order are set by equation (1) in which at least one of coefficients b_n ($n = 1, 2, \dots, N$) is nonzero. In the case, when $b_1 = b_2 = \dots = b_n = 0$, the relation (1) presents the class of the linear nonrecursive digital filters named also as finite impulse response (FIR) filters.

Let us consider the FIR filter with the impulse response $\{h_n = a_n\}_{0 \leq n \leq N}$. According to (1), the current sample $y(l)$ of output signal is computed with the help of the last N samples $x(l), x(l-1), \dots, x(l-N+1)$ of input signal as follows:

$$y(l) = \sum_{n=0}^{N-1} h_n x(l-n) (l = 0, 1, \dots). \quad (3)$$

With respect to property of adaptability, this concept is usually treated as ability to carry out efficiently the partial modification or the full change of impulse response adequately to dynamics of a current situation. In other words, the possibility of the choice of any digital filters from some basic set should be ensured. Thus, the process of the adaptive FIR filtration can be described by equation

$$y(l) = \sum_{n=0}^{N-1} h_u(n)x(l-n), \dots (l = 0, 1, \dots), \quad (4)$$

where $u \in \{0, 1, \dots, U-1\}$; U is the number of elements in the used set of digital filters.

The principal distinctive feature of the offered high speed implementation of convolution (4) consists in application of a new tabular procedure of multiplication of numbers presented in a position code by constants of the fixed set with resulting in the code of minimum redundant MNS.

Let all the samples $x(l-n)$ of input signal be the integer values from some range $\{-P, -P+1, \dots, P\}$ and be represented by the additional binary codes

$$\left(X_{l,n}^{(\lambda-1)} X_{l,n}^{(\lambda-2)} \dots X_{l,n}^{(0)} \right)_2$$

of the length $\lambda = \lceil \log_2(2P+1) \rceil$ bits, where P is a natural number; $X_{l,n}^{(t)} \in \{0, 1\}$; $t = 0, 1, \dots, \lambda-1$; $[x]$ denotes the nearest right integer number to real x . Then it is possible to represent $x(l-n)$ in the following positional form:

$$x(l-n) = \sum_{t=0}^{\lambda-1} X_{l,n}^{(t)} 2^t - X_{l,n}^{\lambda-1} 2^\lambda = \sum_{t=0}^{\lambda-2} X_{l,n}^{(t)} 2^t - X_{l,n}^{\lambda-1} 2^{\lambda-1}. \quad (5)$$

We introduce the notation

$$\begin{aligned} X_{l,n}^{(s)} &= \left(X_{l,n}^{(r_s+\lambda_s-1)} X_{l,n}^{(r_s+\lambda_s-2)} \dots X_{l,n}^{r_s} \right)_2 = \\ &= \sum_{t=0}^{\lambda_s-1} X_{l,n}^{(r_s+t)} 2^t; \quad (s = 0, 1, \dots, \mu-1) \end{aligned} \quad (6)$$

and

$$F_s(X^{(s)}) = \begin{cases} X^{(s)} 2^{r_s}, & \text{if } s = 0, 1, \dots, \mu-2 \\ X^{(\mu-1)} 2^{r_{\mu-1}} - \left\lfloor \frac{X^{(\mu-1)}}{2^{\lambda_{\mu-1}-1}} \right\rfloor 2^\lambda, & \text{if } s = \mu-1, \end{cases} \quad (7)$$

where $r_0 = 0, r_1, r_2, \dots, r_{\mu-1}$ is the ascending sequence of integer values setting a decomposition of a binary code

$$\left(X_{l,n}^{(\lambda-1)} X_{l,n}^{(\lambda-2)} \dots X_{l,n}^{(0)} \right)_2$$

of a sample $x(l-n)$ into $\mu \geq 1$ groups; the s th group contains $\lambda_s = r_{s+1} - r_s$ bits; $r_{\mu-1} \leq \lambda-1, r_\mu = \lambda$; $X^{(s)}$ is an arbitrary element of the set $\{X_{l,n}^{(s)} | l = 0, 1, \dots, n = 0, 1, \dots, N-1\}$, $s = 0, 1, \dots, \mu-1$; $[x]$ denotes the nearest left integer number to real x [5, 9].

In view of equality

$$\left\lfloor \frac{X_{l,n}^{(s)}}{2^{\lambda\mu-1}} \right\rfloor = X_{l,n}^{(\lambda-1)},$$

expression (5) taking into account (6) and (7) is led to a form

$$x(l-n) = \sum_{s=0}^{\mu-1} F_s \left(X_{l,n}^{(s)} \right). \quad (8)$$

Substituting (8) into (4), we will obtain

$$y(l) = \sum_{n=0}^{N-1} \sum_{s=0}^{\mu-1} h(n) F_s \left(X_{l,n}^{(s)} \right), \quad (l = 0, 1, \dots). \quad (9)$$

We approximate the product $h_u(n)F_s(X^{(s)})$ by fractions $\frac{Y_{s,u}(n, X^{(s)})}{S}$, where

$$Y_{s,u}(n, X^{(s)}) = \lfloor S h_u(n) F_s(X^{(s)}) \rfloor; \quad (10)$$

S is the natural scale defining the accuracy of approximation.

Then the process of the adaptive FIR filtration, which is realized in accordance with (9), is represented with satisfactory accuracy by the following approximate relation:

$$y(l) \approx \frac{1}{S} \sum_{n=0}^{N-1} \sum_{s=0}^{\mu-1} Y_{s,u} \left(n, X_{l,n}^{(s)} \right), \quad \dots \quad (l = 0, 1, \dots) \quad (11)$$

Let us note that the sum

$$Y_u(n, l) = \sum_{s=0}^{\mu-1} Y_{s,u} \left(n, X_{l,n}^{(s)} \right), \quad (12)$$

appearing in (11), after division by S gives an approximate value of the product $h_u(n)x(l-n)$ (see (4), (8) and (9)). Thus, equation (12) represents a required basic relation of applied multiplication method. For small $U, N, \mu, \lambda_0, \lambda_1, \dots, \lambda_{\mu-1}$ the values $Y_{s,u}(n, X_{l,n}^{(s)})$ can be obtained by the table method (see (6), (7)), and in any code: positional, modular or another. Therefore, the offered method of multiplication is qualified here as table.

As far as

$$-\frac{1}{2} \leq S h_u(n) F_s \left(X_{l,n}^{(s)} \right) - \left\lfloor S h_u(n) F_s \left(X_{l,n}^{(s)} \right) \right\rfloor < \frac{1}{2},$$

for an error of approximation of the product $h_u(n)F_s(X_{l,n}^{(s)})$ by the fraction $\frac{Y_{s,u}(n, X_{l,n}^{(s)})}{s}$ the estimation

$$-\frac{1}{2S} \leq h_u(n)F_s(X_{l,n}^{(s)}) - \frac{Y_{s,u}(n, X_{l,n}^{(s)})}{s} < \frac{1}{2S} \quad (13)$$

is true.

Adding inequalities (13) termwise for all $s = 0, 1, \dots, \mu - 1$ and taking into account (8) and (12), we obtain

$$-\frac{\mu}{2S} \leq h_u(n)x(l-n) - \frac{Y_u(n,l)}{s} < \frac{\mu}{2S}. \quad (14)$$

Hence the method of multiplication presented by formulas (6) - (8), (10), (12) possesses an error which varies in the interval $[-\frac{\mu}{2S}; \frac{\mu}{2S}]$. Term-by-term summation of inequalities (14) for all $n = 0, 1, \dots, N-1$ with the consequent usage of (4), (11) and (12) evaluates the total error of realizable process of the FIR filtration:

$$-\frac{\mu N}{2S} \leq y(l) - \frac{Y(l)}{s} < \frac{\mu N}{2S}, \quad (15)$$

Where

$$Y(l) = \sum_{n=0}^{N-1} Y_u(n, l). \quad (16)$$

As follows from (14) and (15), the accuracy of filtration depends on values of parameters N , μ and S (see (11)). However, it can be controlled practically only by means of scaling factor S which defines an error of approximation of the products $h_u(n)F_s(X_{l,n}^{(s)})$ by the fractions $\frac{Y_{s,u}(n, X_{l,n}^{(s)})}{s}$. In the context of the proposed solution, the scale S is selected so that firstly a size of changing of integer values $Y_{s,u}(n, X_{l,n}^{(s)})$ (see (10)) was not less than a range $\{-P, -P+1, \dots, P\}$ of input signal $\{x(n)\}$ and secondly that reliability of all resultant samples (11) guarantees not less than in λ most significant digits.

According to (6), (7) and (10), the first of the specified conditions is hold obviously if for every u

$$\max_{n, X_{l,n}^{(\mu-1)}} \left\{ \left| |Sh_u(n)F_{\mu-1}(X_{l,n}^{(\mu-1)})| \right| \right\} = \max_n \{ | | - 2^{\lambda-1} Sh_u(n) | | \} \geq 2^{\lambda-1}$$

or

$$S \max_N \{|h_u(n)|\} > 1$$

Hence, satisfiability of the first condition is ensured if

$$S > (\min_u \{\max_n \{|h_u(n)|\}\})^{-1} \quad (17)$$

For obtaining of the second restriction on a range of the scale S we estimate the value $Y(l)$ (see (16)). From (16), taking into account (12) and (8) - (10), it follows that the upper estimate has the form

$$\begin{aligned} Y(l) &= \sum_{n=0}^{N-1} \sum_{s=0}^{\mu-1} |Sh_u(n)F_s(X_{l,n}^{(s)})| \leq \sum_{n=0}^{N-1} \sum_{s=0}^{\mu-1} \left(Sh_u(n)F_s(X_{l,n}^{(s)}) + \frac{1}{2} \right) = \\ &= \sum_{n=0}^{N-1} \left(Sh_u(n) \sum_{s=0}^{\mu-1} F_s(X_{l,n}^{(s)}) + \frac{\mu}{2} \right) = S \sum_{n=0}^{N-1} h_u(n)x(l-n) + \frac{N\mu}{2} \leq \\ &\leq SP \sum_{n=0}^{N-1} |h_u(n)| + \frac{N\mu}{2}. \end{aligned} \quad (18)$$

Analogously, we obtain the lower estimate of $Y(l)$. Appropriate calculations lead to the inequality

$$Y(l) > - \left(SP \sum_{n=0}^{N-1} |h_u(n)| + \frac{N\mu}{2} \right). \quad (19)$$

As follows from (15),

$$-\frac{1}{2} \leq \frac{Sy(l)-Y(l)}{N\mu} < \frac{1}{2},$$

Therefore

$$\left| \frac{Sy(l)-Y(l)}{N\mu} \right| = 0$$

Consequently, the upper parts $\frac{Y(l)}{N\mu}$ of samples $Y(l)$, which are computed by a rule (16), can be considered as reliable. In order to ensure that the bit capacity of these parts was not less than λ bit, we demand, according to (8) and (19), the fulfillment of the condition

$$\frac{\min_u \left\{ SP \sum_{n=0}^{N-1} |h_u(n)| + \frac{N\mu}{2} \right\}}{N\mu} \geq P.$$

The given inequality is satisfied if

$$S \geq \frac{N\mu(2P-1)}{2P \min_u \{\sum_{n=0}^{N-1} |h_u(n)|\}} \quad (20)$$

From (17) and (20) it follows that the choice of scale S should be made taking into account the inequality

$$S \geq \left[\max \left\{ \frac{1}{\min_u \{\max_n \{|h_u(n)|\}\}}, \frac{N\mu(2P-1)}{2P \min_u \{\sum_{n=0}^{N-1} |h_u(n)|\}} \right\} \right]. \quad (21)$$

Let, for example, the samples of impulse response $\{h_u(n)\}_{0 \leq n \leq N}$ ($u = 0, 1, \dots, U-1$) of realizable filter do not overstep the limits of a segment $[-1, 1]$. Then the minimum possible value of a right-hand member of (21) is $S_{\min} = \mu$. If this value is selected as a required scale ($S = \mu$), then in accordance with (14) and (15) an absolute error of the offered method of multiplication will not exceed a threshold $\frac{1}{2}$, while an absolute error of output samples $\frac{Y(l)}{S}$ will not exceed a threshold $\frac{N}{2}$. Such an accuracy is quite acceptable for digital FIR filters.

Estimations (18) and (19) for $Y(l)$ show that an implementation of the basic calculated relation (11) in minimal redundant MNS [5-7, 9] requires that parameter M must satisfy the requirement

$$M > \left[SP \max_u \{\sum_{n=0}^{N-1} |h_u(n)|\} + \frac{N\mu}{2} \right]. \quad (22)$$

In particular, if $h_u(n) \in [-1, 1]$ and $S = \mu$, then (22) gives the following restrictive condition for M :

$$M > \left[\frac{N\mu(2P+1)}{2} \right]. \quad (23)$$

On the basis of results obtained above, the algorithm of high speed implementation of adaptive FIR filtration in minimal redundant MNS can be synthesized as follows:

1. The number $u \in \{0, 1, \dots, U-1\}$ of impulse response $\{h_u(n)\}_{0 \leq n \leq N}$ of desired filter is specified.
2. Let the initial values $l = 0, Y_0 = 0, Y_1 = 0, \dots, Y_{N-1} = 0$. The values of variables Y_0, Y_1, \dots, Y_{N-1} are represented in minimal redundant MNS with modules m_1, m_2, \dots, m_k .
3. The minimum redundant modular code $(\xi_1^{(l-1)}, \xi_2^{(l-1)}, \dots, \xi_k^{(l-1)})$ of current value Y_0 is converted to an additional binary code $(y_{\lambda'-1}^{(l-1)}, y_{\lambda'-2}^{(l-1)}, \dots, y_0^{(l-1)})_2$ of the length $\lambda' = \lceil \log_2 M \rceil + 1 \text{ bit}$ ($y_t^{(l-1)} \in \{0, 1\}, t = 0, 1, \dots, \lambda' - 1$).

4. Variable n is set as 0, i.e. $n = 0$.

5. Minimal redundant modular codes

$\left(\xi_1^{(l)}(X_{l,0}^{(s)}, n, u), \xi_2^{(l)}(X_{l,0}^{(s)}, n, u), \dots, \xi_k^{(l)}(X_{l,0}^{(s)}, n, u) \right)$ of values

$Y_{s,u}(n, X_{l,0}^{(s)})$ (see (10)) are calculated by means of tabular procedure for every $s = 0, 1, \dots, \mu - 1$ according to a rule

$$\xi_i^{(l)}(X_{l,0}^{(s)}, n, u) = \left| \text{Sh}_u(n) F_s(X_{l,0}^{(s)}) \right|_{m_i} \quad (i = 1, 2, \dots, k) \quad (24)$$

for current sample $x(l)$ of input signal.

6. Digits of modular code $\left(\xi_1^{(l)}(n), \xi_2^{(l)}(n), \dots, \xi_k^{(l)}(n) \right)$ of numerator of the fraction $\frac{Y_u(n,l)}{s}$ approximating the product $h_u(n) x(l)$ are computed according to (12)

$$\xi_i^{(l)}(n) = \left| \sum_{s=0}^{\mu-1} \xi_i^{(l)}(X_{l,0}^{(s)}, n, u) \right|_{m_i} \quad (i = 1, 2, \dots, k). \quad (25)$$

7. The value

$$Y_n = Y_{n+1} + Y_u(n, l) \quad (26)$$

is assigned to variable Y_n .

8. Equality $n = N - 1$ is checked. In the case of its satisfiability, the brunching to the next step 9 is executed. Otherwise, the value n is increased by 1 ($n = n + 1$) and the cyclical process of calculation of variables Y_0, Y_1, \dots, Y_{N-1} continues starting from step 5.

9. The condition $l = L$ is checked, where L is the number of definable samples of output signal. In the case $l < L$ the value l is increased by 1 ($l = l + 1$) and the declared operations is repeated from step 3. In the event when $l = L$, the computation process of required sample $y(0) \approx \frac{Y(0)}{s}, y(l) \approx \frac{Y(l)}{s}, \dots, y(L-1) \approx \frac{Y(L-1)}{s}$ of output signal of the FIR filter with impulse response $\{h_u(n)\}_{0 \leq n \leq N}$ (see (11), (12), (16), (24) – (26)) is completed.

Let us note that when the samples $x(-N + 1), x(-N + 2), \dots, x(-1)$ of input signal are not necessarily equal to zero and can have arbitrary values from a range $\{-P, -P + 1, \dots, P\}$, then in this case at step 2 of algorithm the initial value of variable l is $l = -N + 1$.

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MODULARNE ZASADY BARDZO SZYBKIEJ ADAPTACYJNEJ FILTRACJI SYGNAŁÓW DYSKRETNÝCH

Streszczenie

W artykule przedstawiono bardzo szybką metodę adaptacyjnej filtracji sygnałów dyskretnych, która oparta jest na wykorzystaniu minimalnie nadmiernego kodowania modularnego. Cechą charakterystyczną oferowanej realizacji filtrów *o skończonej odpowiedzi impulsowej* jest zastosowanie nowej metody mnożenia tabelarycznego liczb pozycyjnych przez stałe z uzyskaniem iloczynów w minimalnie nadmiernym systemie modularnym

keywords: cyfrowe przetwarzanie sygnałów, filtry cyfrowe, modularne struktury obliczeniowe, modularne systemy liczbowe, arytmetyka modularna