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SOME REMARKS OF DIFFERENT AGGREGATION MODES APPLICATIONS WITHIN THE FRAMEWORK OF INTUITIONISTIC FUZZY WEIGHTS

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Abstract

In the classical intuitionistic fuzzy sets theory it is known, that the use of all aggregation modes is not always possible, because of the lack of definition of raising intuitionistic fuzzy values to the intuitionistic fuzzy power. The main aim of this work is to introduct an operation of raising of intuitionistic fuzzy values to an intuitionistic fuzzy power, which does not require conversion to intuitionistic fuzzy values. Additionally, we will present a heuristic method of raising an intuitionistic fuzzy values to the intuitionistic fuzzy power and consideration about its properties.

1. Introduction

In the classical intuitionistic fuzzy sets theory it is well known that different variants of the aggregation of local criteria give rise to different results. It follows from the fact, that the validity of the stage of formulation of a global criterion as an aggregation of the local criteria is dominant. It is obvious that the evaluation of validity of the criteria is not essential in some optimization processes and sometimes all local criteria have the same validity (weight) for decision-makers. In addition, the definition of weights by using of real numbers sometimes is not possible. So, in some cases using the transformation of verbal terms to interval or fuzzy values applied to various types of aggregation modes is more accurate. There are many aggregation modes using for decision making. They can be described not only by real numbers [1, 2, 3, 4, 5, 6, 10, 11, 12, 13, 14, 15].

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Intuitionistic fuzzy sets proposed by Atanasov in [3] are one of the most popular generalizations of the fuzzy sets theory. It is used primarily for resolving of Multiple Criteria Decision Making (MCDM) [9, 25, 26, 27, 28, 29, 30] and group MCDM [5, 6, 32, 33, 35, 36, 37, 38] problems in the cases, when value of the local criteria of alternatives and/or their weight are intuitionistic fuzzy values.

Some problems with intuitionistic fuzzy uncertainty in framework of MCDM are based on disadvantages of classical operations defined on intuitionistic fuzzy values. In [7] some limitations of conventional operations on fuzzy values were discussed. The proper critical examples can be found in [19].

In the paper [8] an approach, based on intuitionistic fuzzy matrix and relations between elements of this matrix, which are the intuitionistic fuzzy sets, was proposed. It was proved in [19], that the method of comparing intuitionistic fuzzy numbers proposed in [8] does not always lead to correct results.

The next problem with the classical intuitionistic fuzzy sets theory is the lack of the definition of raising an intuitionistic fuzzy values to the intuitionistic fuzzy power. It is worth noticing that the lack of this definition strongly reduces the number of aggregation modes that we can apply in MCDM problems under condition of intuitionistic fuzzy uncertainty.

It was proved in the paper [19], that the use of Dempster-Shafer theory based on conversion of intuitionistic fuzzy values to belief intervals allows to get more reliable results and simplifies the calculations in the solution of MCDM problem. But for cases when using of the conversion of intuitionistic fuzzy values is not advisable, we propose an heuristic method of raising an intuitionistic fuzzy values to intuitionistic fuzzy power.

The rest of the paper is set up as follows. In Section 2, we show basic definitions of classical intuitionistic fuzzy sets theory. Section 3 is devoted to an extension of intuitionistic fuzzy set theory in framework of Dempster-Shafer theory. In Section 4, we show an operation on intuitionistic fuzzy sets in framework of Dempster-Shafer theory and introduce new operators of raising an intuitionistic fuzzy values to the intuitionistic fuzzy power, and Intuitionistic Fuzzy Weighted Geometric operator with weights is presented by intuitionistic fuzzy values. In section 5, we proved some properties of exponentiation operation realized by transformation to belief intervals and heuristic IFV method.

2. Basic definitions

Intuitionistic fuzzy sets theory proposed by Atanasov [3] is one of the most popular generalizations of the fuzzy sets. It is used primarily for resolving MCDM problems [9, 25, 26, 27, 28, 29, 30] and group MCDM [5, 6, 32, 33, 35, 36, 37, 38] in the cases, when value of the local criteria of alternatives and/or their weight are intuitionistic fuzzy values.

The definition of intuitionistic fuzzy set is based on consideration of membership function μ and non-membership function v of element x to a set X, where $0 \leq \mu(x) + v(x) \leq 1$. Then we can construct a set $\{\langle x, \mu(x), v(x)\rangle : x \in X\}$, where $0 \leq \mu(x) + v(x) \leq 1$. For constant $x \in X$, a pair $\langle \mu(x), v(x)\rangle$ is called intuitionistic fuzzy value (IFV) or intuitionistic fuzzy number. In the next consideration IFV will be described shortly as $\langle \mu, v\rangle$ because for the fixed set X these two functions determine all such values for all $x \in X$.

In [40] and [41] some aggregation operators based on the synthesis of intuitionistic fuzzy sets and Dempster-Shafer Theory (DST) were presented. It is easy to see that operators based on the Choquet integral [41] are useful in situations, where the aggregate weight of the assessments have some correlation with each other.

In the paper [18] the strong link between intuitionistic fuzzy sets and DST was shown. This link allows the direct application of a Dempster's rule of combination in MCDM problems to aggregate local criteria with intuitionistic fuzzy values. The link between IFS and DST was also revealed in [23, 24].

In [3] Atanasov gives the following definition of the intuitionistic fuzzy set:

Definition 1. Let $X = \{x_1, x_2, \ldots, x_n\}$ be a finite universal set. A set A is called intuitionistic fuzzy set (IFS) over the set X if A has the following form: $A = \{\langle x_j, \mu_A(x_j), v_A(x_j)\rangle : x_j \in X\}$, where functions $\mu_A : X \to [0, 1]$ and $v_A : X \to [0, 1]$ determines the degree of membership and non-membership of the element $x_j \in X$, and, for each $x_j \in X$, the inequality $0 \leq \mu_A(x_j) + v_A(x_j) \leq 1$ holds.

A parameter $\pi_A(x) = 1-(\mu_A(x)+v_A(x))$ is called an intuitionistic index (or the hesitation degree) of the element $x \in X$ [3]. Of course, for each $x_j \in X$, we have $0 \leq \pi_A(x) \leq 1$.

An intuitionistic set is the generalization of ordinary fuzzy one, so all of the typical results from the classical fuzzy set theory may be convert in framework of the intuitionistic fuzzy sets theory (IFST). Additionally all researches based on ordinary fuzzy sets can be described by IFS's. On the other hand, IFST contains not only operations compatible on fuzzy sets, but also such operations, that cannot be defined in framework of the ordinary fuzzy set theory [20].

The operations of addition $\oplus$ and multiplication $\otimes$ on IFV are defined shortly by Atanasov [4] as follows:

Let $A = \langle \mu_A, v_A \rangle$ and $B = \langle \mu_B, v_B \rangle$ be IFV's. Then we have:
(1) $A \oplus B = \langle \mu_A + \mu_B - \mu_A\mu_B, v_A v_B \rangle$,
(2) $A \otimes B = \langle \mu_A\mu_B, v_A + v_B - v_A v_B \rangle$.
These operators were constructed in such a way, that the result of its using is IFV too. It is easy to prove that $0 \leq \mu_A + \mu_B - \mu_A\mu_B \leq 1$ and $0 \leq v_A + v_B - v_A v_B \leq 1$.
Based on operations (1) and (2) the following expressions were received in [16] for each integer $n = 1, 2, ...$ We have:
$nA = \underbrace{A \oplus A \oplus \ldots \oplus A}_{n\ -\ times} = \langle 1 - (1 - \mu_A)^n, v_A^n \rangle$ and
$A^n = \underbrace{A \otimes A \otimes \ldots \otimes A}_{n\ -\ times} = \langle \mu_A^n, 1 - (1 - v_A)^n \rangle$.
It was shown that these operations can be used not only for integer values, but also for the real values $\lambda, \lambda_1, \lambda_2 > 0$, i.e.:
(3) $\lambda A = \langle 1 - (1 - \mu_A)^\lambda, v_A^\lambda \rangle$, and (4) $A^\lambda = \langle \mu_A^\lambda, 1 - (1 - v_A)^\lambda \rangle$.
The operations (1)-(4) have the following algebraic properties (see [39]).
Let $A = \langle \mu_A, v_A \rangle$ and $B = \langle \mu_B, v_B \rangle$ be IFV's. Then:
(5) $A \oplus B = B \oplus A$, (6) $A \otimes B = B \otimes A$
(7) $\lambda(A \oplus B) = \lambda A \oplus \lambda B$, (8) $(A \otimes B)^\lambda = A^\lambda \otimes B^\lambda$
(9) $\lambda_1 A \oplus \lambda_2 A = (\lambda_1 + \lambda_2)A$, (10) $A^{\lambda_1} \otimes A^{\lambda_2} = A^{\lambda_1 + \lambda_2}$.
Operations (1)-(4) are used for the aggregation of local criteria in the case of solving the MCDM problems in terms of fuzzy intuitionistic uncertainty.
Let $A_1, A_2, \ldots, A_n$ be an IFV's of local criteria and $w_1, w_2, \ldots, w_n$, $(\sum_{i=1}^n w_i = 1)$ be a weights of this criteria. Then the Intuitionistic Weighted Arithmetic Mean (IWAM) may be specify by using the operation (1) and (3) as follows [18]:
(11) $IWAM = w_1 A_1 \oplus \ldots \oplus w_n A_n = \langle 1 - \Pi_{i=1}^n (1 - \mu_{A_i})^{w_i}, \Pi_{i=1}^n v_{A_i}^{w_i} \rangle$.
The aggregation operator (11) gets the result in the IFV form and it is idempotent. This aggregation operator is the most popular operator for solving MCDM problems under conditions of intuitionistic fuzzy uncertainty. It is also worth noticing that there are no problems with the idempotent Intuitionistic Fuzzy Weighted Geometric operator (IFWG), which can be obtained directly from (2) and (4):
(12) $IFWG = A_1^{w_1} \otimes \ldots \otimes A_n^{w_n} = \langle \Pi_{i=1}^n \mu_{A_i}^{w_i}, 1 - \Pi_{i=1}^n (1 - v_{A_i})^{w_i} \rangle$.

3. An extension of intuitionistic fuzzy set theory in framework of DST

In the paper [18] the close link between intuitionistic fuzzy sets and DST was demonstrated. This link allows the direct application of Dempster's rule of combination in MCDM problems to aggregate local criteria with intuitionistic fuzzy values.

In the paper [18] the possibility of transformation of intuitionistic fuzzy values to Belief Intervals (BI), based on the extension of intuitionistic fuzzy sets theory in the context of DST, was shown as well. This fact allows to present mathematical operations on the IFVs as operations on BI.

Let A be an intuitionistic set over the set X. The set A is treat like a question or proposition, and X is a proposition set, or a set of hypothesis, which exclude each other, or a set of answers [41]. The structure of DST is linked to mapping m, which is called the basic assignment function from subset of X on interval $m : 2^X \to [0,1]$ such that $m(\emptyset) = 0, \sum_A = 1$. Subsets of X, for which this mapping does not assume zero, are called focal elements.

In [34] Shafer introduced several new measures. The belief measure is a mapping $Bel : 2^X \to [0,1]$ such that for any subset B from X occurring the expression [18]:

(13) $\quad Bel(B) = \sum_{i=1}^{n} m(A_i), \quad \emptyset \neq A_i \subseteq B, \quad i = 1, \ldots, n.$

The next measure proposed by Shafer is a measure of plausibility, which is a mapping $Pl : 2^X \to [0,1]$ such that for any subset B from X the relation

(14) $\quad Pl(B) = \sum_{i=1}^{n} m(A_i), \quad A_i \cap B \neq \emptyset, \quad i = 1, \ldots, n$

holds [18].

It is easy to see that $Bel(B) \leq Pl(B)$. A DST allows to show a clear measure of ignorance about the opportunity B and its completion $\overline{B}$ as the length of the interval $[Bel(B), Pl(B)]$. This interval, called belief interval (BI), can be also interpreted as the inaccuracy of the probability of opportunity B [18].

In [25], Hong and Choi proposed an interval representation $[\mu_A(x_j), 1 - v_A(x_j)]$ of IFS A on X instead of a pair $\langle \mu_A(x_j), v_A(x_j) \rangle$ in framework of MCDM problems.

The first obvious advantage of this approach is that the expression $[\mu_A(x_j), 1 - v_A(x_j)]$ represents the real interval with its right bound being not less than left one (due to the rule $0 \leq \mu_A(x_j) + v_A(x_j) \leq 1$). Moreover the second advantage is the consideration of the basic definition of intuitionistic fuzzy sets theory in terms of the DST.

The following definition was proposed in [18].

Definition 2. Let $X = \{x_1, x_2, \ldots, x_n\}$ be an universal finite set and x_j be an element from X represented by functions $\mu_A(x_j), v_A(x_j)$ representing the membership and non-membership of the element $x_j \in X$ with conditions mentioned in Definition 1. Intuitionistic fuzzy set A over X is an object of the following form: $A = \{\langle x_j, BI_A(x_j) : x_j \in X\}$, where $BI_A(x_j) = [Bel_A(x_j), Pl_A(x_j)]$ is a belief interval, and $Bel_A(x_j) = \mu_A(x_j)$, and $Pl_A(x_j) = 1 - v_A(x_j)$ are the belief and plausibility functions of $x_j \in X$ belonging to a set A over X.

At the first glance, the Definition 1 represents a simple re-definition of IFS as an interval fuzzy set. However the semantic of DST allows to increase the reliability of the calculations, when we deal with operations on the IFVs and MCDM problems. In particular, such approach allows aggregation of the local criteria represented by IFVs and development of MCDM method without defuzzification, while the local criteria and their weights are represented by IFVs. As a result, we get a final assessment in the form of belief interval [18].

4. Operations on IFVs in framework of DST

The paper [19] suggests two approaches to formulate the operation on belief intervals. The first one is based on a probability interpretation of belief intervals, while the second one is based on non-probability interpretation. It was proved in [19] that the operations based on the non-probability interpretation of belief intervals have much better algebraic properties than operations based on the probability approach. It is important to point out that both these approaches generate arithmetic operators, which have much better properties than arithmetic operations within the framework of the classical intuitionistic fuzzy sets theory. Therefore, we will use only operations defined in [19] based on non-probability interpretation on belief intervals.

Let $A = \langle \mu_A, v_A \rangle$ and $B = \langle \mu_B, v_B \rangle$ be the IFVs represented by belief intervals $BI(A) = [Bel(A), Pl(A)]$, $BI(B) = [Bel(B), Pl(B)]$, where $Bel(A) = \mu_A$, $Pl(A) = 1 - v(A)$, and $Bel(B) = \mu_B$, $Pl(B) = 1 - v(B)$ respectively. In this case, $Bel(A)$ and $Pl(A)$ are measures of belief and plausibility, such as element $x_j \in X$ belongs to a set A over the set X. The belief interval $BI(A) = [Bel(A), Pl(A)]$ can be treated as an interval belonging to a true power of ascertainment (argument, proposition, hypothesis etc.).

In [19] the additional and multiplication operators on belief intervals are shown. It is possible, when we define additional operator $\oplus_{BNP}$ of belief intervals as follows:

$BI(A) \oplus_{BNP} BI(B) = [\frac{Bel(A)+Bel(B)}{2}, \frac{Pl(A)+Pl(B)}{2}]$.

So, if we have n different ascertainments represented by belief intervals $BI(A_i)$, then their sum can be defined as follows:

$\underbrace{BI(A_1) \oplus_{BNP} \ldots \oplus_{BNP} BI(A_n)}_{n-times} = [\frac{1}{n}\sum_{i=1}^{n} Bel(A_i), \frac{1}{n}\sum_{i=1}^{n} Pl(A_i)]$.

The multiplication operation of belief intervals we can define as follows [19]:

$$BI(A) \otimes_{BNP} BI(B) = [Bel(A)Bel(B), Pl(A)Pl(B)]. \quad (15)$$

It is easy to see, that this operator is the same multiplication one in conventional interval arithmetic [31]. The scalar multiplication is defined in [19] as follows:

(16) $\lambda BI(A) = [\lambda Bel(A), \lambda Pl(A)]$,

where $\lambda \in [0, 1]$ (for $\lambda > 1$ this operator does not always lead to the real belief intervals). This restriction is justified by the fact that we can define operations on belief intervals for MCDM problems, where λ usually represents the weight of local criteria, which are smaller than one.

The exponentiation operation is defined in [19] as follows:

(17) $(BI(A))^{\lambda} = [(Bel(A))^{\lambda}, (Pl(A))^{\lambda}]$.

and it leads to a real belief interval for all $\lambda \geq 0$.

Using the conventional rules of interval arithmetic [33], we obtain $BI(A)^{BI(B)} = [\alpha, \beta]$, where:

$\alpha = min\{Bel(A)^{Bel(B)}, Bel(A)^{Pl(B)}, Pl(A)^{Bel(B)}, Pl(A)^{Pl(B)}\}$, and

$\beta = max\{Bel(A)^{Bel(B)}, Bel(A)^{Pl(B)}, Pl(A)^{Bel(B)}, Pl(A)^{Pl(B)}\}$.

Taking into account the properties of the belief intervals, we can lead these expression to a following form [19]

(18) $BI(A)^{BI(B)} = [Bel(A)^{Pl(B)}, Pl(A)^{Bel(B)}]$.

The operators certain in that way, having good algebraic properties (the same as in the case of the conventional theory of IFSs, see (5) -(10)). It can be directly inferred from expressions (14)-(17):

$BI(A) \oplus BI(B) = BI(B) \oplus BI(A), \quad BI(A) \otimes BI(B) = BI(B) \otimes BI(A),$

$(BI(A) \otimes_{BNP} BI(B))^{\lambda} = BI(A)^{\lambda} \otimes_{BNP} BI(B)^{\lambda},$

$BI(A)^{\lambda_1} \otimes_{BNP} BI(A)^{\lambda_2} = (BI(A))^{\lambda_1+\lambda_2},$

$\lambda BI(A) \oplus_{BNP} \lambda BI(B) = \lambda(BI(A) \oplus_{BNP} BI(B)),$

$\lambda_1 BI(A) \oplus_{BNP} \lambda_2 BI(A) = (\lambda_1 + \lambda_2)(BI(A) \oplus_{BNP} BI(A)).$

Using expressions (14) and (16) we get following Intuitionistic Weighted Arithmetic Mean (IWAM):

(19) $IWAM_{DSTNP}(A_1, \ldots, A_n) = [\frac{1}{n}\sum_{i=1}^{n} w_i Bel_{A_i}, \frac{1}{n}\sum_{i=1}^{n} w_i Pl_{A_i}]$

Observe, that this operator is not idempotent [19]. However, a small modification of (19) (multiplication by n) allows to obtain an idempotent operator:

(20) $IWAMI_{DSTNP}(A_1, \ldots, A_n) = [\sum_{i=1}^{n} w_i Bel_{A_i}, \sum_{i=1}^{n} w_i Pl_{A_i}]$

The Intuitionistic Fuzzy Weighted Geometric operator ($IFWG_{DSTP}$) obtained directly from (12) and (17) has the form [19]:

(21) $IFWG_{DSTP}(A_1, \ldots, A_n) = [\Pi_{i=1}^{n} Bel_{A_i}^{w_i}, \Pi_{i=1}^{n} Pl_{A_i}^{w_i}]$

It is easy to see that the operator (21) is idempotent.

The Intuitionistic Fuzzy Weighted Geometric operator with weights ($IFWGB_{DSTP}$) presented by belief intervals $BI = [Bel_i, Pl_i], i = 1, \ldots, n$, obtained directly from (15) and (17) has the following form [19]:

(22) $IFWGB_{DSTP}(A_1, \ldots, A_n) = [\Pi_{i=1}^{n} Bel_{A_i}^{Pl_i}, \Pi_{i=1}^{n} Pl_{A_i}^{Bel_i}]$

It was shown in [19] that the result obtained by means of this operator has the form of belief intervals. This operator is not idempotent. Of course, the idempotence of the operator (21) is guaranteed by the normalization of weight value in the form of the real numbers, or $\sum_{i=1}^{n} w_i = 1$. Given that in (22) weights are have a belief interval's form $BI_i = [Bel_i, Pl_i]$ we have a problem with their normalization [19].

Using the proposed approach (15) and (20), we get IWAM in the case when the local criteria and their weights are IFVs.

Let $BI_i = [Bel_i, Pl_i], i = 1, \dots, n$, be belief intervals corresponding to the intuitionistic fuzzy weights of the local criteria $A_i, i = 1, \dots, n$ presented by belief intervals $BI(A_i) = [Bel_{A_i}, Pl_{A_i}], i = 1, \dots, n$.

Then, from (15) and (19) we get [19]

(23) $IWAMB_{DSTNP}(A_1, \dots, A_n) = [\frac{1}{n}\sum_{i=1}^{n} Bel_i Bel_{A_i}, \frac{1}{n}\sum_{i=1}^{n} Pl_i Pl_{A_i}]$

The simple modification of foregoing operator (multiplying by n) allows to obtain a more handy operator [19]:

(24) $IWAMB_{DSTNP}(A_1, \dots, A_n) = [\sum_{i=1}^{n} Bel_i Bel_{A_i}, \sum_{i=1}^{n} Pl_i Pl_{A_i}]$

This operator is not idempotent. Of course, the idempotence of operator (24) is guaranteed by the normalization of weight value in the form of the real numbers, or $\sum_{i=1}^{n} w_i = 1$, while in (21) weights have a belief interval's form $BI = [Bel_i, Pl_i], i = 1, \dots, n$.

Among the basic properties of the aggregation operations (boundary conditions, monotonicity, continuity, symmetry, idempotence, and others), idempotence seems to be particularly important in MCDM problems. In conclusion we can say that the operators introduced in the framework of the non-probability treatment of belief interval have their counterparts in the classical theory of IFSs [19].

So, for example despite the fact, that in the classical intuitionistic fuzzy sets theory is no definition of raising an IFV to intuitionistic fuzzy power. Taking into account the analysis of the raise to the power using conversion IFSs to belief intervals we get the following expression:

(25) $A^B = \langle \mu_A^{1-v_B}, 1 - (1 - v_A)^{\mu_B} \rangle.$

Let us consider an example of calculating using the convert to belief intervals and expression (29).

Example 1. Let $A = \langle 0.78, 0.2 \rangle$ and $B = \langle 0.44, 0.33 \rangle$. Then $BI(A) = [0.78, 0.8]$ and $BI(B) = [0.44, 0.67]$, so $BI(A)^{BI(B)} = [0.8467, 0.9065]$. Using expression (25), we get $A^B = \langle 0.8467, 0.9065 \rangle$. It is easy to see, that the results coincide qualitatively, and $BI(A^B) = [0.8467, 0.9065]$ too.

Similarly, their equivalent operator is an operator presented in the following expression:

(26) $A^B = \langle \Pi_{i=1}^{n} \mu_{A_i}^{(1-v_i)}, 1 - \Pi_{i=1}^{n} (1 - v_{A_i})^{\mu_i} \rangle.$

	C_1	C_2	C_3
A_1	(0.780, 0.200, 0.020)	(0.661, 0.190, 0.149)	(0.600, 0.300, 0.100)
A_2	(0.770, 0.200, 0.030)	(0.685, 0.299, 0.016)	(0.650, 0.150, 0.200)
A_3	(0.556, 0.400, 0.044)	(0.459, 0.229, 0.312)	(0.500, 0.200, 0.300)

Table 1. An assessment of alternatives according to the three criteria in the intuitionistic fuzzy form

It was proved in [21] that the approach to the comparison of intervals based on the subtraction operation compartments intervals Δ_{A-B} has the explicit advantages in comparison with other methods.

Then, regardless of whether the belief intervals intersect or not, we have $BI(A) > BI(B)$ in the case when subtraction of interval $BI(A)$ is bigger than subtraction of interval $BI(B)$, or:

(27) $BI(A) > BI(B)$, if $\frac{Bel(A)+Pl(A)}{2} > \frac{Bel(B)+Pl(B)}{2}$.

From the expression (27), we can deduce (see [20]), that $BI(A) > BI(B)$, if

(28) $BI(A) > BI(B)$, if $Bel(A) + Pl(A) > Bel(B) + Pl(B)$.

It is easy to see, that this inequality is an equivalent to the inequality, or at a glance, where and are score functions. If (Bel(A)+Pl(A)) = (Bel(B)+Pl(B)) then BI(A) = BI(B).

5. Comparison of results obtained

In this section we compare the result obtained by aggregation modes in the framework of classical intuitionistic fuzzy approach and approach based on DST.

Consider the results obtained using the expressions on $IWAM$ (11) and $IWAM_{DSTNP}$ (24).

Example 2. Let us consider that in the MCDM problem alternatives are presented by IFV, while the weights of the local criteria are presented by real numbers. Data obtained during the analysis of the various alternatives are presented in the Table 1.

In this example, weights have the following form:

(29) $W = [w_1, w_2, w_3] = [0.33, 0.35, 0.32]$.

It is easy to see that the validity of the criteria in this example are close to each other.

Then, using the expression (11) on additive aggregation $IWAM$, we obtain the results shown in the Table 2.

However, in the case of consideration of this example in the framework of DST, first we need to convert the values shown in table 1 to the value in the form of belief intervals. Modified data are presented in the Table 3.

	μ_{A_i}	v_{A_i}	$S(A_i), i = 1, 2, 3$	Ranking
A_1	0.7142	0.2555	0.4588	1
A_2	0.6314	0.2188	0.4126	2
A_3	0.5730	0.2215	0.3515	3

TABLE 2. Ranking obtained by $IWAM$ aggregation for IFVs Intuitionistic fuzzy assessment

	C_1	C_2	C_3
$BI(A_1)$	[0.780, 0.800]	[0.661, 0.810]	[0.600, 0.700]
$BI(A_2)$	[0.770, 0.800]	[0.685, 0.701]	[0.650, 0.850]
$BI(A_3)$	[0.556, 0.600]	[0.459, 0.771]	[0.500, 0.800]

TABLE 3. Assessment of alternatives according to the three criteria in belief intervals form

	Interval assessment	Numerical assessment	Ranking
A_1	[0.1755, 0.1833]	0.1794	2
A_2	[0.3369, 0.4260]	0.3815	1
A_3	[0.1108, 0.1488]	0.1298	3

TABLE 4. Ranking obtained by $IWAM_{DSTNP}$ aggregation for BI's

Considering the expressions (24) and (2) and the data contained in table 3, we obtain results for $IWAM_{DSTNP}$ presented in the Table 4.

It is easy to see the difference between the results presented in Tables 2 and 4. Such incompatibility we have due to the exchange of the multiplication operator to the exponentiation operator. This is because, among other, the lack of precision of the results obtained by the formula (11) . To the benefit of the second method (24) speaks also using a direct interval extension, the advantages of which are presented in [21].

Example 3. Consider the Example 2 for $IFWGB_{DSTP}$ (22) and its equivalent operator $IFWGI$ (26) for the following intuitionistic fuzzy weights:

$$W = [w_1, w_2, w_3] = \begin{bmatrix} 0.44 & 0.33 & 0.23 \\ 0.46 & 0.32 & 0.22 \\ 0.44 & 0.34 & 0.22 \end{bmatrix}. \tag{30}$$

The weights converted to belief intervals are presented in the expression (31).

$$BI[W] = [BI_1, BI_2, BI_3] = \begin{bmatrix} 0.44 & 0.67 \\ 0.46 & 0.68 \\ 0.44 & 0.66 \end{bmatrix}. \tag{31}$$

	Interval assessment	Numerical assessment	Ranking
A_1	[0.4796, 0.6563]	0.5679	1
A_2	[0.3436, 0.6839]	0.5137	3
A_3	[0.3399, 0.7214]	0.5307	2

TABLE 5. Ranking obtained by aggregation for BI

	μ_{A_i}	v_{A_i}	$S(A_i), i = 1, 2, 3$	Ranking
A_1	0.4796	0.3437	0.1359	1
A_2	0.3436	0.3161	0.0275	3
A_3	0.3399	0.2786	0.0613	2

TABLE 6. Ranking obtained by $IFWGI$ aggregation for IFVs.

	$BI(A_n)$	Numerical assessment	Ranking
A_1	[0.4796, 0.6563]	0.5679	1
A_2	[0.3436, 0.6839]	0.5137	3
A_3	[0.3399, 0.7214]	0.5307	2

TABLE 7. Ranking of belief intervals of $IFWGI$.

	C_1	C_2	C_3
A_1	(0.580, 0.200, 0.220)	(0.651, 0.190, 0.159)	(0.600, 0.300, 0.100)
A_2	(0.670, 0.100, 0.230)	(0.585, 0.299, 0.116)	(0.650, 0.150, 0.200)
A_3	(0.556, 0.400, 0.044)	(0.459, 0.229, 0.312)	(0.500, 0.200, 0.300)

TABLE 8. Assessment of alternatives according to the three criteria in intuitionistic fuzzy form

The results obtained by using $IFWGB_{DSTP}$ (22) and $IFWGI$ (26) are presented in Tables 5 and 6, respectively.

Then calculated by the expression (26) provides results quality equivalent to results obtained by the expression (22), as it shown in Tables 5-7.

It's easy to see, that ranking and number results are the same in case of both aggregation modes. That's indicate that the heuristic method of calculation $IFWGI$ has the same precision as $IFWGI_{DSTNP}$.

Example 4. Consider the results obtained by using the $IFWGB_{DSTNP}$ (22) and $IWAMB_{DSTNP}$ (24). Assume that, in the $MCDM$ problem, alternatives and weights of local criteria are presented by IFV. Data obtained during the analysis of the various alternatives are presented in the Table 8.

In this example, weights have the following form

	C_1	C_2	C_3
$BI(A_1)$	[0.580, 0.800]	[0.651, 0.810]	[0.600, 0.700]
$BI(A_2)$	[0.670, 0.900]	[0.585, 0.701]	[0.650, 0.850]
$BI(A_3)$	[0.556, 0.600]	[0.459, 0.771]	[0.500, 0.800]

TABLE 9. Assessment of alternatives according to the three criteria in belief intervals form

	Interval assessment	Numerical assessment	Ranking
A_1	[0.2047, 0.5137]	0.3592	2
A_2	[0.2599, 0.5173]	0.3886	1
A_3	[0.1983, 0.517]	0.3577	3

TABLE 10. Ranking obtained by $IWAMI_{DSTP}$ for BIs

	Interval assessment	Numerical assessment	Ranking
A_1	[0.3582, 0.7517]	0.5550	1
A_2	[0.3054, 0.5702]	0.4378	3
A_3	[0.3399, 0.6127]	0.4763	2

TABLE 11. Ranking obtained by $IFWGB_{DSTP}$ for BIs

$$(32) \quad W = [w_1, w_2, w_3] = \begin{bmatrix} 0.34 & 0.33 & 0.33 \\ 0.46 & 0.32 & 0.22 \\ 0.44 & 0.34 & 0.32 \end{bmatrix}.$$

Firstly, we need to convert the values shown in the Table 8 to the values in the form of belief intervals. Modified data presented in the Table 9.

Weights converted to belief intervals are presented in the expression (33).

$$(33) \quad BI[W] = [BI_1, BI_2, BI_3] = \begin{bmatrix} 0.34 & 0.67 \\ 0.46 & 0.68 \\ 0.34 & 0.66 \end{bmatrix}.$$

Then, using the formula (24) on additive aggregation $IWAMI_{DST}$, we get the results shown in table 10.

Taking into account the expressions (33) and (22) and the data contained in table 9, we obtain results for presented in table 11.

It is easy to see the difference in the results presented in tables 10 and 11. As it was proved in [17], multiplication aggregation mode is more believable than the additive aggregation mode.

6. Final remarks

In this paper a few usefull methods of solving problems with intuitionistic fuzzy weights in order to be able to use any aggregation modes for decision-making problems are proposed. The first of them is based on a new operator of raising an intuitionistic fuzzy values to intuitionistic fuzzy power and allows to use a classic form of intuitionistic fuzzy sets. The second one is based on Dempster-Shafer theory and modifications of intuitionistic fuzzy sets to belief intervals. This method allows to use an interval arithmetic and direct interval extension, the advantages of which is shown in [21], to solve $MCDM$ problems.

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