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# Assessment of Optimal Bandwidth in Decomposition of Redistribution Coefficient ${ }^{1}$ 

## Introduction

Progressive nature, characterizing most of contemporary income tax systems in developed countries, suggests that tax system is seen as an important instrument of income redistribution. This form of redistribution is, however, one of the most controversial. The first problem is social acceptance: the common consciousness of differences in tax duties sometimes results in sense of unfairness. The second - and more technical - argument against progressive taxation is low efficiency of income redistribution done in this way.

Assessment of redistribution efficiency demands, however, estimation of tax system characteristics, especially redistribution capacity of a given tax schedule. The most popular coefficient, measuring extent of redistribution is given as a difference between concentration indices before and after taxation. It is a measure of effective redistribution, comprising both redistribution resulting from progressive tax scale and redistribution being a consequence of unintended tax inequity. Separation of these two effects - and assessment of a theoretical redistribution capacity of the tax schedule - is possible thanks to decomposition of redistribution coefficient. However, this redistribution involves dividing the whole population into groups with identical (or similar) income. Obtained results suggest strong dependence of decomposition results on the choice of income bandwidth, but there exists no method that enables unambiguous choice of this bandwidth.

In this paper we present analyses of relation between redistribution coefficient values and income bandwidth, and some criteria for choosing this band-

[^0]width. We also propose some other, possible criteria that could be useful in practical applications.

This paper is prepared within a joint research, done in collaboration with prof. Achille Vernizzi, University of Milan. In this place we would like to acknowledge his remarks and suggestions that helped us a lot. Of course, all errors and omissions are ours.

## Redistribution measurement

The basic index, used in redistribution measurement is $R E$ coefficient, defined as follows (cf. 0):

$$
\begin{equation*}
R E=G_{Y}-G_{Y-T} \tag{1}
\end{equation*}
$$

where $G_{Y}$ denotes Gini index for income before taxation and
$G_{Y-T}$ - Gini index for income after taxation.
The value of this coefficient could be interpreted as a percentage of income that is transferred from the richer to the poorer as a result of diversified tax rates. This kind of redistribution does not take the form of direct money transfers. It is a hypothetical value of such transfers that should be made in case of hypothetical, proportional tax system to get the tax distribution identical to the analysed one.

Decomposition of Gini index that forms the basis for construction of the redistribution coefficient, enables to isolate between-group ( $G_{Y}^{B}$ ) and withingroup ( $G_{Y}^{W}$ ) inequality. This property results in possibility of decomposition of $R E$ coefficient. The main goal of this decomposition is an answer to the question: to what extent the overall redistribution is a consequence of intentional construction of the tax system and to what extent is it restricted by tax inequity? The first component could be interpreted as a measure of actual, theoretical redistribution capacity, while the second reflects undesired - and often unintended effects of the taxation.

Generally, decomposition of $R E$ coefficient could be written as

$$
\begin{equation*}
R E=V-H-R \tag{2}
\end{equation*}
$$

where $V$ is a measure of vertical effect (decrease in inequality, resulting from tax system progressivity) and H reflects horizontal inequity (unequal treatment of
equals). Differences in inequality levels, resulting from changes in order of taxpayers with respect to income before and after taxation, are captured by component R.

As mentioned above, calculation of $R E$ coefficient decomposition has to be preceded with a division of the taxpayers' population into groups, distinguished from the point of view of the income.

Let $Y$ be a vector of non-decreasing incomes before taxation for $n$ taxpayers:

$$
Y=\left(y_{1}, y_{2}, \ldots, y_{n}\right), \quad y_{1} \leq y_{2} \leq \ldots \leq y_{n},
$$

and taxpayers are grouped (with respect to income) into $k$ classes, consisting of $n_{1}, n_{2}, \ldots, n_{k}$ taxpayers respectively. Analogously, $Y-T$ would denote incomes after taxation.

There are proposed in the literature several methods of decomposing $R E$ coefficient. The first such a decomposition was described by Kakwani (cf. [2]):

$$
R E=V^{K}-R^{A P K},
$$

where:

$$
\begin{gathered}
V^{K}=G_{Y}-D_{Y-T} \text { (Reynolds-Smolensky redistribution index), } \\
R^{A P K}=G_{Y-T}-D_{Y-T} \text { (Atkinson-Plotnick-Kakwni index), }
\end{gathered}
$$

and
$D_{Y-T}$ is a concentration index for income after taxation, calculated in the same way as Gini index but for data ordered by income before taxation.

Because decomposition assumes division of taxpayers into groups with exactly equal incomes, horizontal effect is equal to zero. It is known, however, that division of population into exact-equal groups is very difficult (or even impossible) in practical applications. Therefore, consecutive decomposition on RE coefficient allows division into groups with similar incomes. Below we present three decomposition methods that take into account such "close-equal" groups.

The first one is a modification of a decomposition proposed by Aronson, Johnson and Lambert (AJL) (cf. 0). AJL method - in the original version - was taking into account groups with exactly equal incomes, but Urban and Lambert (cf. 0) show the possibility of extension on the groups with similar incomes. They introduce smoothed, linear taxation within each group. The rate of this tax is calculated individually for each group as an effective tax rate. Such neutral tax
wipes out redistribution within each group. Then AJL decomposition could be given as (cf. 0):

$$
\begin{equation*}
R E=V^{A J L}-H^{A J L}-R^{A J L}, \tag{3}
\end{equation*}
$$

where:

$$
\begin{gathered}
V^{A J L}=\left(G_{Y}^{B}-G_{Y-T}^{B}\right)-\left(G_{Y-T}^{S W}-G_{Y}^{W}\right), \\
H^{A J L}=G_{Y-T}^{W}-G_{Y-T}^{S W}, \\
R^{A J L}=G_{Y-T}-G_{Y-T}^{B}-G_{Y-T}^{W} .
\end{gathered}
$$

$G_{Y}^{B}$ denotes between-group Gini index for income before taxation, where all individual incomes within each group were replaced with average incomes for a given group. Within-group Gini index $\left(G_{Y}^{W}\right)$ is given by the formula:

$$
\begin{equation*}
G_{Y}^{W}=\sum_{k} \frac{n_{k}}{n} \cdot \frac{n_{k} \bar{Y}_{k}}{n \bar{Y}} \cdot G_{k, Y}, \tag{4}
\end{equation*}
$$

where $G_{k, Y}$ denotes Gini index for $k$-th group, $\bar{Y}_{k}$ - average income in $k$-th group. Measures concerning income after taxation are denoted by $G_{Y-T}^{B}$ and $G_{Y-T}^{W}$ respectively. Within-group Gini index $G_{Y-T}^{S W}$ is calculated in an analogous way as given by (4), but for the income after taxation and smoothed tax. If $y_{1}, y_{2}, \ldots, y_{n_{k}}$ are the incomes in $k$-th group and $t_{1}, t_{2}, \ldots, t_{n_{k}}$ are respective tax amounts, smoothed tax for $i$-th taxpayer (belonging to $k$-th group) is given by

$$
t_{i}^{s}=\frac{\sum_{i=1}^{n_{k}} t_{i}}{\sum_{i=1}^{n_{k}} y_{i}} \cdot y_{i}=g \cdot y_{i} .
$$

Other decomposition method was proposed by van de Ven, Creedy and Lambert (VCL) (cf. 0). It is given by the formula:

$$
\begin{equation*}
R E=V^{V C L}-H^{V C L}-R^{A J L} \tag{5}
\end{equation*}
$$

where:

$$
V^{V C L}=G_{Y}^{B}-G_{Y-T}^{B},
$$

$$
H^{V C L}=G_{Y-T}^{W}-G_{Y}^{W}
$$

Van de Ven, Creedy and Lambert allow decomposition for taxpayers with similar (not necessarily exact) incomes. At the same time they assume - as in the AJL model - that taxation causes no change in order of both taxpayers and groups of taxpayers. The latter means the same order of average incomes (within defined groups of "close-equals") before and after taxation.

The last decomposition method-UL method, presented in this paper, takes the following form (cf. 0):

$$
\begin{equation*}
R E=V-H-R^{A P K} \tag{6}
\end{equation*}
$$

where:

$$
\begin{gathered}
V=V^{V C L}-\left(G_{Y-T}^{S W}-G_{Y}^{W}\right)+\left(G_{Y-T}^{B}-D_{Y-T}^{B}\right) \\
H=D_{Y-T}^{W}-G_{Y-T}^{S W}
\end{gathered}
$$

$D_{Y-T}^{B}$ is between-group and $D_{Y-T}^{W}$ within-group concentration index for income after taxation. Both indices are defined analogously to $G_{Y-T}^{B}$ and $G_{Y-T}^{W}$, but incomes are ordered as if they were incomes before taxation. If taxation causes no change of order, concentration indices $D$ take the same values as respective Gini indices.

Contrary to the earlier mentioned decompositions, UL method takes into account possibility of incomes re-ranking. This change of order could be observed both in case of individual incomes (within one group or even between groups) and in case of whole groups (when average incomes in groups are re-ordered).

## Income bandwidth definition

Decomposition methods, presented in the previous section require division of the whole population into groups of taxpayers with similar income. To this end, suitable bandwidth has to be chosen and all the taxpayers have to be assigned to classes with respect to their income before taxation.

Obtained results strongly depend on the bandwidth $h$. The influence of the choice of the bandwidth on decomposition results (given by formula (5)) is presented in the Figure 1. Calculations were made for bandwidths ranging between 20 PLN and 3000 PLN. With increase of the bandwidth, it could be observed rise in share of $H^{V C L}$ in $R E$ and decrease of $R^{A J L}$ share. Vertical effect $V^{V C L}$ is
on a slight increase. These diversified results suggest necessity of the proper choice of the income bandwidth.

Van de Ven, Creedy and Lambert (cf. 0) propose to use bandwidth that maximises vertical effect $-V$. Taking into account that overall decomposition $(R E)$ could be unambiguously calculated from individual data (it does not depend on decomposition method), maximising $V$ leads to maximum overall tax inequity, resulting from "errors" of tax system ( $H+R-$ cf. formula (2)). At $V$-maximising bandwidth, measure of intended tax progression and redistribution (given in the form of tax schedule) will not be underestimated. Van de Ven, Creedy and Lambert suggest choosing the bandwidth that maximises $\frac{V}{R E}$. However, when this function has more than one maximum or is very irregular, finding an optimal bandwidth could be very troublesome - what is pointed out by Vernizzi and Pellegrino (cf. 0).

Our empirical analyses indicate irregular behaviour of $\frac{V}{R E}$ and problems with finding the global maximum. Therefore, above characterized method of choosing an optimal bandwidth will not be taken into account in the next of this paper.


Figure 1. Relation between bandwidth and VCL decomposition results
Source: own calculations.
Vernizzi and Pellegrino (hereafter VP - cf. 0) recommend using bandwidth that equalises losses in redistribution ("errors" of the tax system) given by for-
mulas (3) (5) and (6). They analyse differences between the redistribution measures in these three decompositions, and recommend choosing the bandwidth that minimises maximal differences $\left|V-V^{V C L}\right|,\left|V-V^{A J L}\right|$ and $\left|V^{V C L}-V^{A J L}\right|$, stating that the highest value among $V, V^{V C L}$ or $V^{A J L}$ for a given bandwidth is no lower than the lowest global maximum.

Finally, VP criterion comes down to the choice of the bandwidth for which $G_{Y-T}^{B}-D_{Y-T}^{B}=G_{Y-T}^{S W}-G_{Y}^{W}$, where $G_{Y-T}^{B}-D_{Y-T}^{B}=R^{E G}$.

However, apart from decompositions of redistribution coefficient mentioned above, others are proposed in the literature. Therefore, the natural question arises, how could be justified choice of the decompositions taken into account.

In authors' opinion, classes consisting of taxpayers with approximately the same income, should be as similar as possible to the classes consisting of taxpayers with exactly equal income. Therefore, characteristics of "close-equals" classes should reflect characteristics of the "exact-equals" classes. It means that within-group inequality should stay at the approximately the same level ( $G_{Y-T}^{S W}-G_{Y}^{W}$ should be minimised).

Moreover, order of average (in classes) incomes after taxation should not be changed ( $R^{E G}=G_{Y-T}^{B}-D_{Y-T}^{B}$ should be possibly small). Figure 2 displays behaviour of both $G_{Y-T}^{S W}-G_{Y}^{W}$ and $R^{E G}$ in relation to the bandwidth chosen. It could be observed that $R^{E G}$ diminishes within increase in income bandwidth. In case of $G_{Y-T}^{S W}-G_{Y}^{W}$ this behaviour is opposite. To minimise values of both $G_{Y-T}^{S W}-G_{Y}^{W}$ and $R^{E G}$ we propose to choose income bandwidth satisfying following conditions:

$$
\begin{gather*}
R^{E G}+\left(G_{Y-T}^{S W}-G_{Y}^{W}\right) \text { is minimal, or }  \tag{7}\\
R^{E G}=G_{Y-T}^{S W}-G_{Y}^{W} . \tag{8}
\end{gather*}
$$

The second proposal is identical to VP criterion.


Figure 2. Relation between bandwidth, $R^{E G}$ and $G_{Y-T}^{S W}-G_{Y}^{W}$
Source: own calculations
Moreover, on the basis of empirical investigation, described in the next section, we observed that bandwidths resulting from the VP criterion and criteria given by formulae (7) and (8) are very close to each other. We also observed that this bandwidth could be approximated by the formula:

$$
\begin{equation*}
h^{M e M o}=\frac{M e-M o}{10} \tag{9}
\end{equation*}
$$

where Me and Mo denote median and mode for income before taxation respectively. This formula seems to be valid only for Polish data, but for different subsets of taxpayers. It has two main advantages over criteria presented earlier. Firstly, this method requires no extensive calculations. Secondly, this criterion (such as criteria given by (7) and (8)) does not depend on choice of the decomposition method.

Empirical results concerning application of the characterized criteria are presented in the next section.

## Empirical analysis

The empirical part of this paper is based on the tax data from two LowerSilesian revenue offices. This data concern fiscal year 2001 and contain informa-
tion on income and tax paid. In the analysis we distinguished 4 groups of individual taxpayers, taking into account kind of tax form and place of residence:

- Group 1 - individual taxpayers, living in Wrocław and filling form PIT-37 (standard sources of income),
- Group 2 - individual taxpayers, living in Wrocław and filling form PIT-36 (income from own business),
- Group 3 - individual taxpayers, living in Wałbrzych and filling form PIT-37 (standard sources of income),
- Group 4 - individual taxpayers, living in Wałbrzych and filling form PIT-36 (income from own business).
For each group, optimal bandwidth - in the sense of three criteria, given by (7), (8) and (9) - was calculated. Then, for these bandwidths, decomposition results were assessed. Results are presented in Tables 1-4.

Table 1. Decomposition results for taxpayers from Group 1

| Bandwidth | Decomposition of redistribution coefficient |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $R E=V^{\text {AJL }}-H^{\text {AJL }}-R^{\text {AJL }}$ |  |  |  |
|  | RE | $V^{\text {AJL }}$ | $H^{A J L}$ | $R^{\text {AJL }}$ |
| $R^{E G}=G_{y-t}^{S W}-G_{y}^{W}$ | $\begin{gathered} 0,013149 \\ 100 \% \end{gathered}$ | $\begin{gathered} \hline 0,013462 \\ 102,38 \% \end{gathered}$ | $\begin{gathered} 0,000101 \\ 0,77 \% \end{gathered}$ | $\begin{gathered} 0,000212 \\ 1,61 \% \end{gathered}$ |
| $\operatorname{Min}\left(R^{E G}+G_{y-t}^{S W}-G_{y}^{W}\right)$ | $\begin{gathered} 0,013149 \\ 100 \% \end{gathered}$ | $\begin{gathered} \hline 0,013462 \\ 102,38 \% \end{gathered}$ | $\begin{gathered} 0,000084 \\ 0,64 \% \end{gathered}$ | $\begin{gathered} \hline 0,000229 \\ 1,74 \% \end{gathered}$ |
| $\frac{M e-M o}{10}$ | $\begin{gathered} 0,013149 \\ 100 \% \end{gathered}$ | $\begin{gathered} 0,013459 \\ 103,36 \% \end{gathered}$ | $\begin{gathered} 0,000138 \\ 1,05 \% \end{gathered}$ | $\begin{gathered} 0,000172 \\ 1,31 \% \end{gathered}$ |
| x | $R E=V^{V C L}-H^{V C L}-R^{A J L}$ |  |  |  |
|  | RE | $V^{V C L}$ | $H^{V C L}$ | $R^{\text {AJL }}$ |
| $R^{E G}=G_{y-t}^{S W}-G_{y}^{W}$ | $\begin{gathered} 0,013149 \\ 100 \% \end{gathered}$ | $\begin{gathered} 0,013464 \\ 102,40 \% \end{gathered}$ | $\begin{gathered} 0,000103 \\ 0,79 \% \end{gathered}$ | $\begin{gathered} 0,000212 \\ 1,61 \% \end{gathered}$ |
| $\operatorname{Min}\left(R^{E G}+G_{y-t}^{S W}-G_{y}^{W}\right)$ | $\begin{gathered} 0,013149 \\ 100 \% \end{gathered}$ | $\begin{gathered} 0,013464 \\ 102,40 \% \end{gathered}$ | $\begin{gathered} 0,0000855 \\ 0,66 \% \end{gathered}$ | $\begin{gathered} \hline 0,000229 \\ 1,74 \% \end{gathered}$ |
| $\frac{M e-M o}{10}$ | $\begin{gathered} 0,013149 \\ 100 \% \end{gathered}$ | $\begin{aligned} & 0,013465 \\ & 102,40 \% \end{aligned}$ | $\begin{gathered} 0,000143 \\ 1,09 \% \end{gathered}$ | $\begin{gathered} 0,000172 \\ 1,31 \% \end{gathered}$ |

Table 1. Decomposition results for taxpayers from Group 1 (cont.)

| Bandwidth | Decomposition of redistribution coefficient |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $R E=V-H-R^{A P K}$ |  |  |  |
|  | $R E$ | $V$ | $H$ | $R^{A P K}$ |
| $R^{E G}=G_{y-t}^{S W}-G_{y}^{W}$ | 0,013149 | 0,013464 | $-0,000001$ | 0,000316 |
|  | $100 \%$ | $102,40 \%$ | $0,00 \%$ | $2,40 \%$ |
| $\operatorname{Min}\left(R^{E G}+G_{y-t}^{S W}-G_{y}^{W}\right)$ | 0,013149 | 0,013465 | $-0,000000$ | 0,000316 |
|  | $100 \%$ | $102,40 \%$ | $0,00 \%$ | $2,40 \%$ |
| $\frac{M e-M o}{10}$ | 0,013149 | 0,01346 | $-0,000004$ | 0,000316 |
| $2,40 \%$ |  |  |  |  |

Source: own calculations
Table 2. Decomposition results for taxpayers from Group 2

| Bandwidth | Decomposition of redistribution coefficient |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $R E=V^{A J L}-H^{A J L}-R^{A J L}$ |  |  |  |
|  | RE | $V^{\text {AJL }}$ | $H^{\text {AJL }}$ | $R^{\text {AJL }}$ |
| $R^{E G}=G_{y-t}^{S W}-G_{y}^{W}$ | $\begin{gathered} 0,062576 \\ 100 \% \end{gathered}$ | $\begin{gathered} 0,063042 \\ 100,74 \% \end{gathered}$ | $\begin{gathered} 0,000153 \\ 0,24 \% \end{gathered}$ | $\begin{gathered} 0,000313 \\ 0,50 \% \end{gathered}$ |
| $\operatorname{Min}\left(R^{E G}+G_{y-t}^{S W}-G_{y}^{W}\right)$ | $\begin{gathered} 0,062576 \\ 100 \% \end{gathered}$ | $\begin{gathered} 0,063042 \\ 100,74 \% \end{gathered}$ | $\begin{gathered} 0,000127 \\ 0,20 \% \end{gathered}$ | $\begin{gathered} 0,000338 \\ 0,54 \% \end{gathered}$ |
| $\frac{M e-M o}{10}$ | $\begin{gathered} 0,062576 \\ 100 \% \end{gathered}$ | $\begin{gathered} 0,063052 \\ 100,76 \% \end{gathered}$ | $\begin{gathered} 0,000158 \\ 0,25 \% \end{gathered}$ | $\begin{gathered} 0,000317 \\ 0,51 \% \end{gathered}$ |
| x | $R E=V^{V C L}-H^{V C L}-R^{A J L}$ |  |  |  |
|  | RE | $V^{V C L}$ | $H^{V C L}$ | $R^{\text {AJL }}$ |
| $R^{E G}=G_{y-t}^{S W}-G_{y}^{W}$ | $\begin{gathered} 0,062576 \\ 100 \% \end{gathered}$ | $\begin{gathered} 0,063067 \\ 100,79 \% \end{gathered}$ | $\begin{gathered} 0,000178 \\ 0,29 \% \end{gathered}$ | $\begin{gathered} 0,000313 \\ 0,50 \% \end{gathered}$ |
| $\operatorname{Min}\left(R^{E G}+G_{y-t}^{S W}-G_{y}^{W}\right)$ | $\begin{gathered} 0,062576 \\ 100 \% \end{gathered}$ | $\begin{aligned} & 0,063057 \\ & 100,77 \% \end{aligned}$ | $\begin{gathered} 0,000143 \\ 0,23 \% \end{gathered}$ | $\begin{gathered} 0,000338 \\ 0,54 \% \end{gathered}$ |
| $\frac{M e-M o}{10}$ | $\begin{gathered} 0,062576 \\ 100 \% \end{gathered}$ | $\begin{gathered} 0,063078 \\ 100,80 \% \end{gathered}$ | $\begin{gathered} 0,000185 \\ 0,29 \% \end{gathered}$ | $\begin{gathered} 0,000317 \\ 0,51 \% \end{gathered}$ |

Table 2. Decomposition results for taxpayers from Group 2 (cont.)

| Bandwidth | Decomposition of redistribution coefficient |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $R E=V-H-R^{A P K}$ |  |  |  |
|  | $R E$ | $V$ | $H$ | $R^{A P K}$ |
| $R^{E G}=G_{y-t}^{S W}-G_{y}^{W}$ | 0,062576 | 0,063067 | $-0,000006$ | 0,000498 |
|  | $100 \%$ | $100,78 \%$ | $-0,01 \%$ | $0,079 \%$ |
| $\operatorname{Min}\left(R^{E G}+G_{y-t}^{S W}-G_{y}^{W}\right)$ | 0,062576 | 0,063073 | $-0,000001$ | 0,000498 |
|  | $100 \%$ | $100,79 \%$ | $0,00 \%$ | $0,79 \%$ |
| $\frac{M e-M o}{10}$ | 0,062576 | 0,063075 | 0,000001 | 0,000498 |
| $0,79 \%$ |  |  |  |  |

Source: own calculations
Table 3. Decomposition results for taxpayers from Group 3

| Bandwidth | Decomposition of redistribution coefficient |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $R E=V^{A J L}-H^{A J L}-R^{A J L}$ |  |  |  |
|  | RE | $V^{\text {AJL }}$ | $H^{\text {AJL }}$ | $R^{\text {AJL }}$ |
| $R^{E G}=G_{y-t}^{S W}-G_{y}^{W}$ | $\begin{gathered} 0,011044 \\ 100 \% \end{gathered}$ | $\begin{gathered} 0,011319 \\ 102,49 \% \end{gathered}$ | $\begin{gathered} 0,000074 \\ 0,67 \% \end{gathered}$ | $\begin{gathered} 0,000201 \\ 1,82 \% \end{gathered}$ |
| $\operatorname{Min}\left(R^{E G}+G_{y-t}^{S W}-G_{y}^{W}\right)$ | $\begin{gathered} 0,011044 \\ 100 \% \end{gathered}$ | $\begin{gathered} 0,01132 \\ 102,51 \% \end{gathered}$ | $\begin{gathered} 0,000062 \\ 0,57 \% \end{gathered}$ | $\begin{gathered} 0,000214 \\ 1,94 \% \end{gathered}$ |
| $\frac{M e-M o}{10}$ | $\begin{gathered} 0,011044 \\ 100 \% \end{gathered}$ | $\begin{gathered} 0,011321 \\ 102,51 \% \end{gathered}$ | $\begin{gathered} 0,000053 \\ 0,48 \% \end{gathered}$ | $\begin{gathered} 0,000224 \\ 2,03 \% \end{gathered}$ |
| x | $R E=V^{V C L}-H^{V C L}-R^{A J L}$ |  |  |  |
|  | RE | $V^{V C L}$ | $H^{V C L}$ | $R^{\text {AJL }}$ |
| $R^{E G}=G_{y-t}^{S W}-G_{y}^{W}$ | $\begin{gathered} 0,011044 \\ 100 \% \end{gathered}$ | $\begin{aligned} & 0,01132 \\ & 102,50 \% \end{aligned}$ | $\begin{gathered} 0,000075 \\ 0,68 \% \end{gathered}$ | $\begin{gathered} 0,000201 \\ 1,82 \% \end{gathered}$ |
| $\operatorname{Min}\left(R^{E G}+G_{y-t}^{S W}-G_{y}^{W}\right)$ | $\begin{gathered} 0,011044 \\ 100 \% \end{gathered}$ | $\begin{aligned} & 0,011321 \\ & 102,51 \% \end{aligned}$ | $\begin{gathered} 0,000063 \\ 0,57 \% \end{gathered}$ | $\begin{gathered} 0,000214 \\ 1,94 \% \end{gathered}$ |
| $\frac{M e-M o}{10}$ | $\begin{gathered} 0,011044 \\ 100 \% \end{gathered}$ | $\begin{gathered} 0,011321 \\ 102,51 \% \end{gathered}$ | $\begin{gathered} 0,000054 \\ 0,48 \% \end{gathered}$ | $\begin{gathered} 0,000224 \\ 2,03 \% \end{gathered}$ |

Table 3. Decomposition results for taxpayers from Group 3 (cont.)

| Bandwidth | Decomposition of redistribution coefficient |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $R E=V-H-R^{A P K}$ |  |  |  |
|  | $R E$ | $V$ | $H$ | $R^{A P K}$ |
| $R^{E G}=G_{y-t}^{S W}-G_{y}^{W}$ | 0,011044 |  |  |  |
|  | 0,01132 | $-0,000002$ | 0,000279 |  |
| $2,53 \%$ |  |  |  |  |
| $\operatorname{Min}\left(R^{E G}+G_{y-t}^{S W}-G_{y}^{W}\right)$ | 0,011044 |  |  |  |
|  | $100 \%$ | 0,011322 | $-0,000001$ | 0,000279 |
| $\frac{M e-M o}{10}$ | 0,011044 | 0,011323 | 0,000000 | 0,000279 |
| $100 \%$ | $102,53 \%$ | $0,00 \%$ | $2,53 \%$ |  |

Source: own calculations
Table 4. Decomposition results for taxpayers from Group 4

| Bandwidth | Decomposition of redistribution coefficient |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $R E=V^{\text {AJL }}-H^{\text {AJL }}-R^{\text {AJL }}$ |  |  |  |
|  | $R E$ | $V^{\text {AJL }}$ | $H^{\text {AJL }}$ | $R^{\text {AJL }}$ |
| $R^{E G}=G_{y-t}^{S W}-G_{y}^{W}$ | $\begin{gathered} 0,029943 \\ 100 \% \end{gathered}$ | $\begin{aligned} & 0,030445 \\ & 101,68 \% \end{aligned}$ | $\begin{gathered} 0,000229 \\ 0,77 \% \end{gathered}$ | $\begin{gathered} 0,000273 \\ 0,91 \% \end{gathered}$ |
| $\operatorname{Min}\left(R^{E G}+G_{y-t}^{S W}-G_{y}^{W}\right)$ | $\begin{gathered} 0,029943 \\ 100 \% \end{gathered}$ | $\begin{gathered} 0,030451 \\ 101,70 \% \end{gathered}$ | $\begin{gathered} 0,000197 \\ 0,66 \% \end{gathered}$ | $\begin{gathered} 0,000311 \\ 1,04 \% \end{gathered}$ |
| $\frac{M e-M o}{10}$ | $\begin{gathered} 0,029943 \\ 100 \% \end{gathered}$ | $\begin{aligned} & 0,030445 \\ & 101,68 \% \end{aligned}$ | $\begin{gathered} 0,000229 \\ 0,77 \% \end{gathered}$ | $\begin{gathered} 0,000273 \\ 0,91 \% \end{gathered}$ |
| x | $R E=V^{V C L}-H^{V C L}-R^{A J L}$ |  |  |  |
|  | $R E$ | $V^{V C L}$ | $H^{V C L}$ | $R^{\text {AJL }}$ |
| $R^{E G}=G_{y-t}^{S W}-G_{y}^{W}$ | $\begin{gathered} 0,029943 \\ 100 \% \end{gathered}$ | $\begin{gathered} 0,030463 \\ 101,73 \% \end{gathered}$ | $\begin{gathered} 0,000246 \\ 0,82 \% \end{gathered}$ | $\begin{gathered} 0,000273 \\ 0,91 \% \end{gathered}$ |
| $\operatorname{Min}\left(R^{E G}+G_{y-t}^{S W}-G_{y}^{W}\right)$ | $\begin{gathered} 0,029943 \\ 100 \% \end{gathered}$ | $\begin{aligned} & \hline 0,030462 \\ & 101,73 \% \end{aligned}$ | $\begin{gathered} \hline 0,000208 \\ 0,69 \% \end{gathered}$ | $\begin{gathered} 0,000311 \\ 1,04 \% \end{gathered}$ |
| $\frac{M e-M o}{10}$ | $\begin{gathered} 0,029943 \\ 100 \% \end{gathered}$ | $\begin{gathered} 0,030463 \\ 101,73 \% \end{gathered}$ | $\begin{gathered} 0,000246 \\ 0,82 \% \end{gathered}$ | $\begin{gathered} 0,000273 \\ 0,91 \% \end{gathered}$ |

Table 4. Decomposition results for taxpayers from Group 4 (cont.)

| Bandwidth | Decomposition of redistribution coefficient |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $R E=V-H-R^{A P K}$ |  |  |  |
|  | $R E$ | $V$ | $H$ | $R^{A P K}$ |
| $R^{E G}=G_{y-t}^{S W}-G_{y}^{W}$ | 0,029943 | 0,030463 | $-0,000015$ | 0,000535 |
|  | $100 \%$ | $101,74 \%$ | $-0,05 \%$ | $1,79 \%$ |
| $\operatorname{Min}\left(R^{E G}+G_{y-t}^{S W}-G_{y}^{W}\right)$ | 0,029943 | 0,030469 | $-0,000009$ | 0,000535 |
|  | $100 \%$ | $101,76 \%$ | $-0,03 \%$ | $1,79 \%$ |
| $\frac{M e-M o}{10}$ | 0,029943 | 0,030463 | $-0,000015$ | 0,000535 |
|  | $100 \%$ | $101,74 \%$ | $-0,05 \%$ | $1,79 \%$ |

Source: own calculations

For each analysed group, maximum difference between expected redistribution $(V)$, calculated for different decomposition methods, equaled to about 0,03 percent point.

Moreover, in most cases estimate for $V$ do not depend on criterion, applied in order to find an optimal bandwidth. Slightly higher - but also very small - differences are observed in case of estimates for horizontal and re-ranking effect. However, they have no real significance in the process of assessment of redistribution loss, resulting from "errors" in tax system.

## Conclusions

Our results suggest that decomposition of redistribution coefficient strongly depends on the income bandwidth, so it is crucial to appropriately assess this interval. However, all analysed criteria for choosing this optimal bandwidth seem to give similar results - decomposition result do not change much when other criteria are being applied.

## Literature

[1] Aronson J.P., Jenkins P., Lambert P.J., Redistributive effect and unequal income tax treatment, "The Economic Journal", 1994,104, 262-270.
[2] Kakwani N.C. (1984), Welfare rankings of income distributions, "Advances in Econometrics", 1984, 3, 191-213.
[3] Urban I., Lambert P.J., Redistribution, horizontal inequity and reranking: how to measure them properly, "Public Finance Review". 2008 (forthcoming).
[4] Van de Ven J., Creedy J., Lambert P.J., Close equals and calculation of the vertical, horizontal reranking effects of taxation, "Oxford Bulletin of Economics and Statistics",2001, 63, 381-394.
[5] Vernizzi A., Pellegrino S., On the Aronson-Johnson-Lambert decomposition of the redistributive effect, "DEAS, Universita degli Studi di Milano", 2007, WP 2007-13.
[6] Vernizzi A., Pellegrino S., On determing "close equals groups" in decomposing redistributive and reranking effects, 2008, mimeo.

## Streszczenie

## Wyznaczanie szerokości przedziałów dochodowych w dekompozycji współczynnika redystrybucji

Najczęściej analizowany współczynnik redystrybucji, wyrażony jako różnica w koncentracji dochodu przed i po opodatkowaniu, stanowi miarę efektywnej redystrybucji, obejmującej zarówno redystrybucję wynikajaccą z progresywnej konstrukcji systemu podatkowego, jak i redystrybucję stanowiącą konsekwencję niezamierzonej niesprawiedliwości opodatkowania. Rozdzielenie tych komponentów umożliwia dekompozycja współczynnika redystrybucji, zaproponowana przez Kakwaniego a następnie wielokrotnie analizowana i modyfikowana przez innych autorów. Przeprowadzone badania wskazuja jednak, że jednym z elementów istotnie wpływających na wyniki dekompozycji jest wybór szerokości przedziałów klasowych dla dochodu. Jednocześnie w literaturze brak jest jednoznacznych wyników odnośnie optymalnego wyboru tej szerokości. W tym kontekście celem artykułu jest prezentacja wybranych kryteriów wyboru szerokości przedziału oraz propozycja innych, możliwych rozwiązań w tym obszarze. Przedstawione zagadnienia zilustrowane zostały wynikami analiz, przeprowadzonych na danych podatkowych, pochodzących z wybranych urzędów skarbowych Dolnego Śląska.

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