Scientific Issues
Jan Długosz University
in Częstochowa
Mathematics XXII (2017)
129–139
DOI http://dx.doi.org/10.16926/m.2017.22.10

TABULAR MINIMAL REDUNDANT MODULAR STRUCTURES FOR FAST AND HIGH-PRECISION COMPUTATIONS USING GENERAL-PURPOSE COMPUTERS

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Abstract

The present paper is a continuation of research in parallel information processing based on the tabular modular computing structures. We deal with the methodology of using a minimal redundant modular number system for high-speed and high-precision computation by means of modern universal multicore processors. Advantages of formal computing mode on the base of modular arithmetic are demonstrated by the example of implementation of digital signal processing procedures. The additive and additive-multiplicative formal computing schemes with the obtained estimations of the cardinality of working ranges for the realization of calculations are presented in the article.

1. Introduction

In recent years, the tabular methods of digital information processing, both at the hardware and software levels, have been widely used to solve the problems of performance improvement, flexible organization of adaptive operating modes and some others in modern computer algebra and arithmetic, as well as in their numerous applications in such fields of science and technology as digital signal processing (DSP), image recognition and image processing, various purpose identification systems, artificial intelligence systems, information protection and similar systems [1-6].

However, within a framework of classical computer-aided algorithmic foundations based on the arithmetic of positional number systems (PNS) there are a number of objective factors that impede intensification of the development and practical implementation of tabular computer structures.

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First of all, such factors include the sequential internal nature of PNS which appears in the presence of interdigit carry propagation during the execution of arithmetic operations that most often does not allow us to carry out an efficient decomposition of the positional computer structures (PCS) into the components acceptable for tabular implementation. Thus, the principles of tabular information processing and the PNS arithmetic totally contradict each other. Taking the above into account, the research aimed to eliminate the indicated contradictory situation seems to be very topical.

The analysis of modern developing information technologies from the point of view of their suitability for the organization of tabular adaptive computing indicates that the modular technology provides the widest range of possibilities in this respect [7-10]. This is caused by the unique property of modular number system (MNS) to perform in a natural way the decomposition of computational processes into the independent or quasi-independent subprocesses determined on mathematical models whose facilities have dimensions many times smaller (by the number of bases) in comparison with a very large dynamic range of the applied MNS.

In modern applications of modular arithmetic (MA) the development of high-performance parallel systems, which operate completely in the so-called formal computing mode (FCM), occupies a highly important place [7, 8]. This mode is characterized by the absence of rounding on the modular segments of the computational processes, i.e. on segments consisting only of modular operations: addition, subtraction and multiplication in the MNS without overflow check. At the same time it is assumed that the computation results on each modular segment do not exceed the limits of the used dynamic range. Since the arithmetic operations in the modular code (MC) are carried out independently for each of the modules, then in the framework of the FCM a high performance is achieved due to the parallel nature of MA along with the absence of rounding.

In recent years, the modular information-processing systems (MIPS) are mostly applied to the implementation of high-precision and absolutely exact computations. First of all, this concerns the applications in the field of DSP, pattern recognition and image processing as well as in information protection. Currently, among the systems of the specified class the priority positions are occupied by the MIPS which can be implemented programmatically, i.e. without the use of special hardware. Within the framework of existing and intensively developing computer technologies of parallel processing the facilities to implement the FCM at the program level and to increase its operating limits are steadily expanding.

2. The computer-arithmetic basis of tabular technologies of modular information processing

At present, the modular direction in computer science, digital information processing and in many other fields of science and technology experiences a stage of rapid development directly linked with the outstanding achievements of integral technologies. During the past 15-20 years, the fundamental and applied researches both on the theory and the use of modular computing structures (MCS) have been focused on the implementation of the main advantages of MNS, first of all those which are caused by tabular nature of MA. In so doing, the following problems have received primary attention:

- the design of special purpose very-large-scale-integration (VLSI) chips, which take into account the nature of MNS as much as possible;
- the optimization of non-modular MCS;
- the numerous particular applications of MA.

Varied and comprehensive electronic devices and components including extensive classes of VLSI memories have been developed for the tabular MCS and special processors on their basis. There are numerous publications of well-known specialists in the field of MSC with the specific examples of modular processing units, processors and systems of tabular type. These examples shows clearly that a family of modular VLSI architectures and chips allows us to implement the majority of single-step computing procedures such as convolution and correlation of discrete sequences, adaptive finite impulse response (FIR) filters, mapping of discrete signals into the spaces of orthogonal projections and similar tasks. Within the framework of existing electronic components and circuits the ability of MNS-based devices to flexible modification of their own architecture is implemented simply by reprogramming of the used VLSI chips. At the same time, the design and implementation of reconfigurable high-performance modular processors and digital information-processing systems functioning entirely in the FCM usually requires too much hardware resources, and in the case of multi-step basic procedures it becomes impossible.

Because of the widespread distribution and rapid development of modern computers the software-based tabular MIPS presents an attractive competitive alternative to the traditional modular computing technologies oriented to the primary use of parallel pipeline VLSI architectures.

In terms of the basic optimality criteria of high-speed MCS a minimal redundant modular arithmetic (MRMA) is used as a computer-arithmetic basis of parallel tabular MIPS oriented to universal multi-processor computers. It is known that the most distinctive feature of an MRMA is the more complete and perfect computer arithmetic in comparison with the analogues of classical MNS [7, 8 11-17]. The software-based versions of MRMA allow us to apply a huge dimension tables (up to 2^{32} words or more), very large modules (for example, from 2^{16} to 2^{64}) and therefore the super-large operating ranges. In the computer models of information processing procedures this circumstance makes it possible to expand significantly the FCM computational limits inherent to conventional modular systems based on VLSI architectures. In turn, this leads to the performance improvement as well as to the considerable increase in the accuracy of the final results.

As is easily seen, the perfect compliance of the MRMA and the tabular principles of the digital information processing appears most clearly in the software-based MCS. First of all, it is expressed in the triviality and flexibility of the programmable substitution mechanism of used operating tables including modification (increase or decrease) and even a complete replacement of the current set of the bases of minimal redundant MNS (MRMNS). Thus, the tabular minimal redundant MCS (MRMCS) and, therefore, the computer models of MIPS based on them have an extremely high level of adaptability and flexibility.

The design of the basic set of computer algorithms and their software models on the grounds of MRMA and modern computers is an inherent development stage of the family of tabular MRMCS, which create a computing environment with the maximum allowable limits of the FMC.

3. The basic model of the computing environment for the technology of modular information processing

The analysis of modern principles and methods used to design efficient algorithms for digital information processing allows us to conclude that usually all of them are aimed at implementing the decomposition concept of computation process organization.

Within the framework of this strategy it is expected the decomposition of the performed computational process (for example, the different discrete transforms, calculation of the convolution, correlation of discrete signals, etc.) into a set of similar procedures of the dimension smaller than the original one, and the construction of a corresponding rule for the formation of a net result by combining all the partial results of the executable procedures. Since most of the frequently used algorithms of digital information processing actually have the same operational structure, then their computer implementation fits into a framework of the common mathematical model.

The digital information processing systems, which target functions are described by the calculating relationships possessing the modular operational spectrum, represent the main application domain for tabular MIPS.

First, let us consider the principles of implementation of the non-recursive (one-step) processes in the FCM (for example, FIR filtering procedures). In this case, the basic elementary computational procedure fits into the framework of generalized computer model which can be described by the following expression

$$Y(l) = \left\rfloor \sum_{n=0}^{N-1} h_{l,n}^{(u)} X_l\,(n) \right\lfloor \ (l = 0, 1, \ldots, L-1;\ u = 0, 1, \ldots, U-1)\,, \quad (1)$$

where $h_{l,n}^{(u)}$ is a certain real constant; $X_l\,(n)$ and $Y\,(l)$ are the integer samples of the input $\{X_l\,(0)\,, X_l\,(1)\,, \ldots, X_l\,(N-1)\}$ and output $\{Y\,(0)$,$Y\,(1)$, $\ldots$,$Y\,(L-1)\}$ digital signals, respectively; N, L and U are the natural numbers; $\rfloor x \lfloor$ designates the nearest integer to the real number x determined by the rule

$$\rfloor x \lfloor = \begin{cases} \lfloor x \rfloor & \text{if } x < \lfloor x \rfloor + 0,5; \\ \lceil x \rceil & \text{if } x \geq \lfloor x \rfloor + 0,5; \end{cases}$$

where $\lfloor x \rfloor = \max\{y \in \mathbf{Z} \mid y < x\}$, $\lceil x \rceil = \min\{y \in \mathbf{Z} \mid y \geq x\}$, $\mathbf{Z}$ is the set of all integers.

Since the expression (1) should be implemented using the MRMA, then it is necessary to reduce it to the form that is consistent with the principle of formal calculations. This can be made in two ways:

a) by the replacement of the coefficients $h_{l,n}^{(u)}$ by the rational fractional approximations $H_{l,n}^{(u)}/Q$, where $H_{l,n}^{(u)} = \left\rfloor Q\, h_{l,n}^{(u)} \right\lfloor \in \{-Q, -Q+1, \ldots, Q-1\}$, Q is the natural number which determines the accuracy of the approximation of real constants $h_{l,n}^{(u)} \in [-1, 1]$ by simple fractions $H_{l,n}^{(u)}/Q$;

b) by the approximation of the products $h_{l,n}^{(u)}\, X_l\,(n)$ by some fractional-rational estimations $Y_l^{(u)}\,(n)\,/S$, where

$$Y_l^{(u)}\,(n) = \left\rfloor S\, h_{l,n}^{(u)}\, X_l\,(n) \right\lfloor\,, \quad (2)$$

S is the selected natural scale. In the first case, the expression (1) is transformed into an approximate model

$$Y(l) \approx \left\rfloor Q^{-1} \sum_{n=0}^{N-1} H_{l,n}^{(u)} X_l\,(n) \right\lfloor \quad (3)$$

and into a model

$$Y(l) \approx \left\rfloor S^{-1} \sum_{n=0}^{N-1} Y_l^{(u)}\,(n) \right\lfloor \quad (4)$$

in the second case ($l = 0, 1, \ldots, L-1$; $u = 0, 1, \ldots, U-1$).

The implementation models (3) and (4) of the expression (1) are called the additive-multiplicative (AM) and additive (A) models, respectively. Thus, the generalized computer model is essentially reduced to computing the sum value

$$Y^{(u)}(l) = \begin{cases} \sum\limits_{n=0}^{N-1} H_{l,n}^{(u)} X_l(n) & \text{in the case of AM-model,} \\ \\ \sum\limits_{n=0}^{N-1} Y_l^{(u)}(n) & \text{in the case of A-model.} \end{cases} \tag{5}$$

The use of FCM is justified only when the computational results obtained on the selected modular segments of the realizable procedure do not exceed the working range of MRMNS. Thus, in order to implement the expression (5) entirely in the FCM it is required that the dynamic range $\mathbf{D} = \mathbf{Z}_{2M}^{-} = \{-M, -M+1, \ldots, M-1\}$ of the default MRMNS should include all the possible values of $Y^{(u)}(l)$ (here, $M = m_0 M_{k-1}$; m_0 is an auxiliary module satisfying the following conditions: $m_0 \geq \rho$ and $m_k \geq 2m_0 + \rho$ [7,8,11]; $M_{k-1} = \prod_{i=1}^{k-1} m_i$; $m_1, m_2, \ldots, m_k$ are the basic modules, $k \geq 2$). This circumstance exerts a decisive influence on the choice not only of the dynamic range $\mathbf{D}$, but also of the source data range $\hat{\mathbf{D}} = \mathbf{Z}_{2P}^{-} = \{-P, -P+1, \ldots, P-1\} \subset \mathbf{D}$ (P is some natural number).

It is assumed that the constant factors from the set $\{H_{l,0}^{(u)}, H_{l,1}^{(u)}, \ldots, H_{l,N-1}^{(u)}\}$ ($l = 0, 1, \ldots, L-1$; $u = 0, 1, \ldots, U-1$), that appeared in (3), are represented in the minimal redundant modular code (MRMC), i.e., by the vectors $H_{l,n}^{(u)} = \left(\left|H_{l,n}^{(u)}\right|_{m_1}, \left|H_{l,n}^{(u)}\right|_{m_2}, \ldots, \left|H_{l,n}^{(u)}\right|_{m_k} \right)$, and the values $X_l(n)$, which in the general case are represented by λ-bits complement binary numbers $X_l(n) = \left(x_l^{(\lambda-1)}(n)\, x_l^{(\lambda-2)}(n) \ldots x_l^{(0)}(n) \right)_2$ ($x_l^{(j)}(n) \in \{0, 1\}$, $j = 0, 1, \ldots, \lambda-1$; $\lambda = 1 + \lceil \log_2 P \rceil$), are transformed into the MRMC ($|X_l(n)|_{m_1}$, $|X_l(n)|_{m_2}$, …, $|X_l(n)|_{m_k}$) during the implementation of the model (3).

With regard to the model (4), for its implementation it is necessary to obtain the MRMC ($\left|Y_l^{(u)}(n)\right|_{m_1}, \left|Y_l^{(u)}(n)\right|_{m_2}, \ldots, \left|Y_l^{(u)}(n)\right|_{m_k}$) of numbers $Y_l^{(u)}(n)$ representing the scaled values of the numbers $X_l(n)$ (see (2)). This can be carried out using the methodology presented in [8].

Therefore, the residues $\left|Y^{(u)}(l)\right|_{m_i}$ ($i = 0, 1, \ldots, k$) of the MRMC of the number $Y^{(u)}(l)$ is calculated according to the rule

$$\left|Y^{(u)}\left(l\right)\right|_{m_i}=\begin{cases}\left|\sum\limits_{n=0}^{N-1}\left|\left|H_{l,n}^{(u)}\right|_{m_i}\left|X_l\left(n\right)\right|_{m_i}\right|_{m_i}\right|_{m_i} & \text{in the case of AM-model,}\\ \left|\sum\limits_{n=0}^{N-1}\left|Y_l^{(u)}\left(n\right)\right|_{m_i}\right|_{m_i} & \text{in the case of A-model}\end{cases} \tag{6}$$

for the implementation of the expression (5) in a FCM.

The tabular data

$$\mathbf{H}_u=\left\{\left(\left|H_{l,n}^{(u)}\right|_{m_1},\left|H_{l,n}^{(u)}\right|_{m_2},\ldots,\left|H_{l,n}^{(u)}\right|_{m_k}\right)\right\}_{n=\overline{0,N-1};\,l=\overline{0,L-1}} \tag{7}$$

are the constituent elements of the computer AM-models (6) of the expression (1). The possibility of programmable partial modification or complete change of the set of MRMC (7) provides a simple adaptive reconfiguration of the computation process. In principle, the number U of the sets $\mathbf{H}_u$ can be an arbitrary number, and the total amount of default constants for tabular information processing is $V_T=N\cdot L\cdot U$ words of length $\lambda_{MNS}=\sum\limits_{i=1}^{k}\lceil\log_2 m_i\rceil$ bits.

Now let us estimate the cardinality $2M$ of the MRMS operating range $\mathbf{D}$ which ensures the correctness of the application of the expression (6).

According to (5), when using the AM-model we have

$$\left|Y^{(u)}\left(l\right)\right|\leq\sum_{n=0}^{N-1}\left|H_{l,n}^{(u)}X_l\left(n\right)\right|\leq P\sum_{n=0}^{N-1}\left|H_{l,n}^{(u)}\right|$$

Therefore, in this case the parameter M of the range $\mathbf{D}$ should satisfy the condition

$$M\geq P\max_{l,u}\left\{\sum_{n=0}^{N-1}\left|H_{l,n}^{(u)}\right|\right\} \tag{8}$$

Without loss of generality, we can assume that $\left|h_{l,n}^{(u)}\right|\leq 1$ for all admissible values of n, l and u. Then, in view of the fact that $H_{l,n}^{(u)}\in\{-Q,-Q+1,\ldots,Q-1\}$ and

$$\max_{l,u}\left\{\sum_{n=0}^{N-1}\left|H_{l,n}^{(u)}\right|\right\}=N\cdot Q$$

we have from (8)

$$M\geq N\cdot P\cdot Q \tag{9}$$

In the case of applying the A-model to the implementation of the expression (1), the necessary estimates for the basic parameter M of the MRMNS range were obtained in [14]:

$$M > \left\lceil \frac{N \cdot S \cdot (2P+1)}{2} \right\rceil .$$

For most modern DSP applications the quite acceptable values of P (a parameter of the data range $\hat{\mathbf{D}}$) and N (a number of digital signal samples) are $2^{16} \leq P \leq 2^{24}$ and $2^8 \leq N \leq 2^{10}$, respectively. Since the calculation of $Y^{(u)}(l)$ (see (5)) in the MRMNS is carried out exactly, i.e. without rounding or truncation errors, then for approximation of the coefficients $H_{l,n}^{(u)}$ we can choose a natural number Q of the size, for example, between 12 and 16 bits, which corresponds to $2^{11} < Q < 2^{16}$. Taking the preceding into account, for the parameter M of the MRMNS operating range we obtain from (9) the following estimate: $2^{35} < M < 2^{50}$.

It is easy to see that implementing FCM within the framework of AM-model, an arbitrary recursive computational process (for example, the Winograd Fourier transform algorithm) represents a set of typical procedures (segments) of the form

$$Y_r^{(l)} = \sum_{n=0}^{N_r-1} C_{r,n,l} X_{r,l}(n) \ \ (r = 0, 1, \ldots, R-1;\, l = 0, 1, \ldots, L_r - 1), \quad (10)$$

where r is the specific number of process stage; R, L_r, N_r are some natural numbers; $C_{r,n,l}$ are the constants from the set $\{-Q, -Q+1, \ldots, Q-1\}$; $\{Y_r(l)\}_{l=\overline{0,L_r-1}}$ and $\{X_{r,l}(n)\}_{n=\overline{0,N_r-1}}$ represent the input and output signals of the rth standard segment of computational process under consideration, respectively. The input signal $\{X_{r,l}(n)\}_{n=\overline{0,N_r-1}}$ is formed from the samples of the input signal $\{X(n)\}_{n=\overline{0,N_r-1}}$ ($N = N_0$ is the signal length) in the case $r = 0$, and from the samples of the output signal $\{Y_{r-1}(l)\}_{l=\overline{0,L_{r-1}-1}}$ if $0 < r < R-1$. The signal $\{Y_{R-1}(l)\}_{l=\overline{0,L_{R-1}-1}}$ on the last stage represents the output signal $\{Y(l)\}_{l=\overline{0,L-1}}$ (L is a natural number) of the basic computational process. In this case, of course, the equality $L_{R-1} = L$ holds.

In the case when the modular computational process has a recursive organization the absolute values of the samples of the output signals of implementable basic procedures steadily increase together with an increase of the number of iterations (see (10)). This leads to the fact that even for a relatively small number of iterations R the cardinality $|\mathbf{D}| = 2M$ of the dynamic range $\mathbf{D}$ of an MRMNS, which ensures the application correctness of the principle of formal calculations, becomes a very large number as it follows from the estimate of the parameter M:

$$M > P \cdot Q^R \cdot \prod_{r=0}^{R-1} N_r. \tag{11}$$

It should be noted that if the samples of the input signal $\{X(n)\}_{n=\overline{0,N-1}}$ and the constants $C_{r,n,l}$ appearing in (10) are integer complex numbers, then the right-hand side of the inequality (11) increases with an extra 2^R.

Therefore, the use of appropriate MRMNS for hardware implementation of recursive computational processes of the type (11) is associated with significant hardware costs and at present it is hardly advisable. At the same time, the tabular MIPS based on the universal processors allows us to realize in full measure the fundamental advantages of MRMA due to the possibility to use the extremely large dynamic ranges $\mathbf{D}$ and to ensure the maximum operation limits of the FCM.

4. Conclusions

With the wide distribution and rapid development of modern computing machinery, the considered approach to creating parallel digital information processing systems on the basis of MRMA is a very convenient and advanced alternative in comparison with other approaches based on the primary use of special purpose digital hardware.

Two types of computer MRMA-models of adaptive DSP procedures that fit into the so-called additive and additive-multiplicative formal computing schemes are presented in the article. The proposed models provide an exceptionally wide range of possibilities for flexible regulation of the working limits of the FCM depending on the current values of the accuracy and other parameters of the computational processes. This makes it possible to select optimally the bases of the required MRMNS for the preset parameters: the cardinality of the operating range, the number of bases, etc.

The possibility of using large dynamic ranges, when implementing the fast and high precision DSP-algorithms by means of the FCM in MRMA, allows us to remove from the general computational process the scaling operations following the standard (elementary) procedure of A- or AM-model. It provides:

- a significant performance increase due to a sharp reduction in the number of non-modular operations;
- the minimization of the upper threshold (and in many cases the total absence) of the computational error;
- the possibility of using a minimal basic set of non-modular operations, i.e. a set that includes only code transformations.

The inclusion of the complex, quadratic and polynomial-scalar variants of MRMA, which are oriented to extra-large dynamic ranges, into the computer-arithmetic base of the technology of tabular MIPS will increase significantly the computational speed (according to preliminary estimates, approximately 4-10 times) on the sets of complex numbers and polynomials. At the same time, the total amount of internal memory for storing the calculated tables is significantly reduced (at least by 40%) in comparison with non-redundant analogs of quadratic and polynomial-scalar MA.

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Received: May 2017

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