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EXPECTED VOLUMES OF REQUESTS IN SYSTEMS OF THE QUEUEING NETWORK WITH A LIMITED NUMBER OF WAITING PLACES

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Abstract

We present a method of finding the expected volume of requests in HM-network with homogeneous requests and bypass of the queueing systems of requests. The case was considered when the volume changes associated with the transitions between the states of the network are deterministic functions, depending on the state of the network and time, and the systems are single line. It is assumed that the probability of the states of the network systems, the parameters of the entrance flow of the requests and the service depend on the time.
Keywords: *HM-networks, queueing network, single-line queueing system, queueing system, queuing time, limited queue, demands total volume, volume of requests, capacity of claims, wireless access point*

1. Introduction

In the information system (IS) the total number of memory volume is bounded by some value, which is usually called memory volume [1]. In the IS designing the main task is the determination of the memory volume so as to take into account the conditions that limit the proportion of the lost information. One of the methods for solving problems of IS design is the use of HM-queueing networks [2]. Further under the IS we will understand systems, converting objects which is the information coming in portions in the form of messages [1]. HM-networks can be used to determine the volume of a buffer storage of systems, which are representing processing IS nodes and transferring messages. Note that considered problem is one of

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the main, for example, in the design of communication centers or hubs in data communication networks.

Neglect time dependence of messages processing of their volumes can lead to serious errors in finding the buffer memory in IS and in calculating of the probability of messages loss. The solution in the general case the above problems can be based on the use of HM-networks with revenues. In such networks, the request during the transition from one queueing system (QS) to another brings some revenue last (which is equal to the volume of this request), and revenue (volume) of the first QS is reduced by this number.

QS with revenues in the stationary regime has been introduced in the consideration in [3], and networks - in [4]. A survey results obtained by queueing systems and queueing networks (QN) in the stationary regime contained in [5]. It is dedicated to finding the mean revenues in the network systems which depend only on their states and do not depend on time, and solving the problem of finding the optimal request service intensities in QS by the method of dynamic programming. QN with revenues in the non-stationary regime has been studied in [6, 7]. Revenues from transitions between network states were depended on the states and their time or were random variables with the given moments of the first and second orders. In a survey article [7] the results of researching, optimization and selection of the optimal strategies in Markov networks with revenues, various applications are described them as a probabilistic models for forecasting expected revenues in the information and telecommunication systems and networks, when, for example, requests service on the server generates revenue for a servicer, as well as insurance companies, logistics transportation systems, industrial systems and other facilities. For the first time application of the HM-networks for estimating the memory volume in the IS has been described in [8].

2. Computer network model with limited number of simultaneous connections

Consider the action model a wireless network. Let us assume that the network is a set of wireless access points S_i each of which gives the user the ability to connect to the network of information by the next available port and use its resources, $i = \overline{1,n}$. Each access point can simultaneously connect multiple users to a network or execute queries users (i.e. for users to make bandwidth sharing for m_i smaller teams). All other requests users do not receive service, create maximum queue requests L_i and are waiting for a reply access point $L_i < \infty$, $i = \overline{1,n}$. From a technical point of view, the value of L_i is determined taking into account the technical characteristics of a wireless access point (WAP). For example, the greater the number of divided bands of the original panel is more customers may be combined, and the slower the speed of the bands assigned to them. The main limitation is the number of possible IP addresses that are distributed to connect.

We also assume that in the absence free places in the queue, to the access point comes the loss of user requests and redirecting this question to the next WAP. From a practical point of view, redirecting associated with direct customer flow (ie, the device working with computer network), or search for «distant» WAPs do not go beyond the radius of visibility of the device.

At request, we assume a data packet, the sending by the source (ie. client device) and tending to the recipient (by WAP). In computer networks, the package is designed in a way, a block of data sent over the network in batch mode. Computer lines, which does not support packet-mode such as traditional telecommunication point-to-point transfer data simply as a sequence of bytes, characters, or bits individually. The package consists of two types of data: the control information and user data (also called a payload). The control information contains information necessary for the provision of user data: source and destination addresses, error detection codes (such as checksums) and information about the order. Typically, the control information contained in the packet header and tail, and the user data disposed therebetween.

Various communication protocols use different conventions for separating elements and formatting data. Protocol packets "Synchronous binary" formats in 8-bit bytes, and for the separation of the used special characters. In other protocols, such as Ethernet header and the beginning of data elements, their location in relation to the package they are registered. Some protocols format the information at the level of bits and not bytes. In this case, it is assumed that each data packet sent by a user will be deterministic or random length (volume).

Many networks cannot guarantee delivery, no duplicates packages, and order delivery, such as the UDP protocol on the Internet. However, this can be done on top of a transport packet (for one level of the OSI model), which can provide such protection. Packet header identifies the type of data packets, the package, the total number of packets and IP address of the source and destination. In our case, it should be assumed that the request sent by the user will always be delivered to the recipient (AP), which will not be taken into account in case of packet loss between the source and the destination.

Estimation of the total volume of data (data packets) for each wireless access point (WAP) at the given point in time is an important task when designing a wireless network, because it allows to locate the highly loaded AP and distribute the load evenly over them.

Therefore, you must determine the average total volume of data (packets of users) received by the access points to the network information (for example, Internet), taking into account the limited number of «simultaneous» connections on this point. This problem can be solved by using HM-network storage service. According to the queueing network we mean a collection of interconnected queueing systems S_i with a limited buffer size L_i to hold the queue (queues) package requests. By application we mean the user's request to the WAP, which is a data packet.

3. Analysis of queueing network with bypass of queueing systems and time depending parameters of entrance flow and service time

Consider the open exponential queueing network comprising n queueing systems. With some probability requests from have a chance to join the queue, or instantly pass in the matrix of transition probabilities to the other queueing system or leave the network. The probability of attachment to the queueing systems depend on the state and number of queueing system from which the notification is sent.

The message is sent to the network at a given time interval $[t, t + \Delta t]$ with probability $\lambda(t)\Delta t + o(\Delta t)$ and is supported in queueing system S_i in this interval with a probability of $\mu_i(t)\Delta t + o(\Delta t)$.

Let p_{ij} - the probability of transition of requests after servicing from the system S_i to S_j, $i, j = \overline{0, n}$, system S_0 we also understand the external environment. Now consider the case where the stream parameters and service depend on the time. Notification is sent from the external environment in the i-th SOM with the probability of p_{0i}, $\sum_{i=0}^{n} p_{0i} = 1$. A request to the queuing system from outside or from the second system at time t with probability $f^{(i)}(k, t)$, when the network is in state of (k, t), connects to the queue, and the probability of $1 - f^{(i)}(k, t)$ not join the requests queue, counts on service (i.e. the time service with a probability of 1 is equal to zero). If the request ended service in the i-th queueing system, then with the probability of p_{ij} is immediately sent to the j-th system and with the probability p_{i0}leaves the queueing network, $\sum_{j=0}^{n} p_{0i} = 1$, $i = \overline{1, n}$.

State of the network is described vector $k(t) = (k, t) = (k_1(t), \ldots, k_n(t)) = (k_1, \ldots, k_n, t)$, where $k_i(t)$ the number of requests in the system S_i (in a queue and handling), $i = \overline{1, n}$. Let $\varphi_i(k, t)$ - conditional probability that the request comes from outside and to the i-th queueing system at time t, when the network is in state (k, t) cannot be handled by queueing system; $\psi_{ij}(k, t))$ - conditional probability that request by introducing point i-th system the outside, at time t, when the network is in state (k, t), for the first time will receive service in the j-th system; $\alpha_i(k,t)$ - conditional probability that requests serviced at time t in the i-th queueing system, when the network is to state (k,t), will not be supported further in either a single system; $\beta_{ij}(k,t)$ - conditional probability that the request serviced in the i-th system at time t, when the network is to state (k,t), for the first time will be supported in the j-th queueing system, $i,j=\overline{1,n}$.

The formula for total probability we get:

$$\varphi_i(k,t) = \left(1 - f^{(i)}(k,t)\right)\left(p_{i0} + \sum_{j=1}^{n} p_{ij}\varphi_j(k,t)\right), i=\overline{1,n},$$

$$\psi_{ij} = f^{(i)}(k,t)\delta_{ij} + \left(1 - f^{(i)}(k,t)\right)\sum_{l=1}^{n} p_{il}\psi_{lj}(k,t), i=\overline{1,n}.$$

If the function $f^{(i)}(k,t) = f^{(i)}(k_i,t)$ depends only on the number of requests, then

$$\varphi_i(k,t) = \left(1 - f^{(i)}(k,t)\right)\left(p_{i0} + \sum_{j=1}^{n} p_{ij}\varphi_j(k,t)\right), i=\overline{1,n}, \tag{1}$$

$$\psi_{ij} = f^{(i)}(k_i,t)\delta_{ij} + \left(1 - f^{(i)}(k,t)\right)\sum_{l=1}^{n} p_{il}\psi_{lj}(k,t), i,j=\overline{1,n}. \tag{2}$$

Furthermore,

$$\alpha_i(k,t) = p_{i0} + \sum_{j=1}^{n} p_{ij}\varphi_j(k-I_i,t),$$

$$\beta_{ij}(k,t) = \sum_{l=1}^{n} p_{il}\psi_{lj}(k-I_i,t), i,j=\overline{1,n}. \tag{3}$$

We also have equality

$$\varphi_i(k,t) + \sum_{j=1}^{n}\psi_{ij}(k,t) = 1,\ i,j=\overline{1,n} \tag{4}$$

$$\alpha_i(k,t) = 1 - \sum_{j=1}^{n}\beta_{ij}(k,t) = 1,\ i,j=\overline{1,n}. \tag{5}$$

From (1) and (3) we find

$$\psi_{ij}(k,t) = f^{(i)}(k,t)\delta_{ij} + \left(1 - f^{(i)}(k,t)\right)\beta_{ij}(k-I_i,t),\ i,j=\overline{1,n}. \tag{6}$$

4. The system of difference-differential equations for the expected volume of requests in the HM-systems network

Let $v_i(k,t)$ - expected volume of requests, which accumulate in the system S_i at time t, when, during the initial network is to state k, and assume that the function is differentiable with respect to t; $r_i(k)$ - increasing the volume of requests in the system S_i per unit time when the network is to state k; $r_{0i}(k+I_i,t)$ - volume of requests, which increases the total volume of requests in the system S_i, when the network passes from the state of (k,t) in to state $(k+I_i,t+\Delta t)$ in time Δt; $-R_{i0}(k-I_i,t)$ - reducing the volume size of the system, if the network causes a transition from state (k,t) in to state $(k+I_i,t+\Delta t)$; $r_{ij}(k+I_i-I_j,t)$ - the volume of requests of the system S_i (reducing the volume of requests of the system S_j), to which increases the total volume of requests, the network changes its state from (k,t) to $(k+I_i-I_j,t+\Delta t)$ in time Δt, $i,j=\overline{1,n}$. Note that we consider now the case when the value of r_{0i}, R_{i0}, r_{ij} are deterministic functions dependent states of network and time.

Assume that the network is in a state (k,t). At time Δt it may be in a state k or go to the states $(k-I_i)$, $(k+I_i)$, $(k+I_i-I_j)$, $i,j=\overline{1,n}$. If the network is still in the state $(k,t+\Delta t)$, volume of requests in the system S_i is $r_i(k)\Delta t$ plus volume $v_i(k,t)$, to which increases its volume in the remaining t time units. The probability of this happening is $1-\sum_{i=1}^{n}\left(\lambda(t)p_{0i}\left(1-\varphi_i(k,t)\right)+\mu_i(t)\left(1-\beta_{ii}(k,t)\right)\right)\Delta t+o(\Delta t)$. If the network passes to the state of $k+I_i,t+\Delta t)$ with probability $\lambda(t)p_{0i}\psi_{ij}(k+I_i,t)\Delta t+o(\Delta t)$, then the total volume of requests in the system S_i is $[r_{0i}(k+I_i,t)+v_i(k+I_i,t)]$ and if the state $(k-I_i,t+\Delta t)$ with probability $\mu_i(k,t)\alpha_i(k-I_i,t)u(k_i,t)\Delta t+o(\Delta t)$, then the total volume of requests of this system will be $[-R_{i0}(k-I_i,t)+v_i(k-I_i,t)]$, $i=\overline{1,n}$.

Similarly, if the network goes from (k,t) in to a state $(k+I_i-I_j,t+\Delta t)$ with probability $\mu_j(k,t)\beta_{ji}(k+I_i-I_j,t)u(k_j,t)\Delta t+o(\Delta t)$, then it will increase the total volume of requests in the system S_i by the amount $r_{ij}(k+I_i-I_j,t)$ plus the volume of the remaining time, if the initial state of the network was $(k+I_i-I_j$. Then, using the formula of the total probability for the expected volume of requests in the system, you can get a system of difference-differential equations (DDE) [9].

$$\frac{\mathrm{d}\nu_i(k,t)}{\mathrm{d}t} = r_i(k) - \sum_{i=1}^{n}\left[\lambda(y)p_{0i}\left(1-\varphi_i(k,t)\right) + \mu_i(t)\left(1-\beta_{ii}(k,t)\right)\right]\nu_i(k,t) + \tag{7}$$

$$+\sum_{j=1}^{n}[\lambda(t)p_{0j}\psi_{ij}(k+I_j,t)\nu_i(k+I_j,t) + \tag{8}$$

$$+\mu_j(t)\alpha_j(k-I_j,t)u(k_j(t))\nu_i(k-I_j,t)] + \tag{9}$$

$$+\sum_{\substack{j=1\\ j\neq i}}^{n}[\mu_j(t)\beta_{ji}(k+I_i-I_j,t)u(k_j(t))\nu_i(k+I_i-I_j,t) + \tag{10}$$

$$+\mu_i(t)\beta_{ij}(k-I_i+I_j,t)u(k_i(t))\nu_i(k-I_i+I_j,t)] + \tag{11}$$

(12)

$$+\sum_{\substack{j=1\\ j\neq i}}^{n}[u_j(t)\beta_{ji}(k+I_i-I_j,t)u(k_j(t))r_{ij}(k+I_i-I_j,t) - \tag{13}$$

$$-u_i(t)\beta_{ij}(k-I_i+I_j,t)u(k_i(t))r_{ji}(k-I_i+I_j,t)] + \tag{14}$$

$$+\sum_{\substack{c,s=1\\ c,s\neq i}}^{n}\mu_s(t)\beta_{sc}(k+I_c-I_s,t)u(k_s(t))\nu_i(k+I_c-I_s,t) + \tag{15}$$

$$+\lambda(t)p_{0i}\psi_{ij}(k+I_i,t)r_{0i}(k+I_i,t) - \tag{16}$$

$$-\mu_i(t)\alpha_i(k-I_i,t)u(k_i(t))R_{i0}(k-I_i,t).$$

Expressions for conditional probabilities $\varphi_i(k,t)$, $\psi_{ij}(k,t)$, $\alpha_i(k,t)$, $\beta_{ij}(k,t)$, $i,j=\overline{1,n}$, originate from (1)-(5).

For a closed network, ie. if $\lambda(t) = 0$, $p_{0i} = p_{i0} = 0$, $r_{0i}(k_i, t) = R_{i0}(k_i, t) = 0$, $\sum_{i=1}^{n} k_i(t) = K$, $i = \overline{1, n}$, then we have from (7)

$$\begin{aligned}
\frac{d\nu_i(k,t)}{dt} &= r_i(k) - \sum_{i=1}^{n} \mu_i(t)\left(1 - \beta_{ii}(k_i,t)\right)\nu_i(k,t) + \\
&+ \sum_{i=1}^{n} \mu_j(t)\alpha_j(k - I_j, t)u(k_j(t))\nu_i(k - I_j, t) + \\
&+ \sum_{\substack{j=1 \\ j \neq i}}^{n} \mu_j(t)\beta_{ji}(k + I_i - I_j, t)u(k_j(t))\nu_i(k + I_i - I_j, t) + \\
&+ \mu_i(t)\beta_{ij}(k - I_i + I_j, t)u(k_i(t))\nu_i(k - I_i + I_j, t) + \\
&+ \sum_{\substack{j=1 \\ j \neq i}}^{n} [\mu_j(t)\beta_{ji}(k + I_i - I_j, t)u(k_j(t))r_{ij}(k + I_i - I_j, t) - \\
&- \mu_i(t)\beta_{ij}(k - I_i + I_j, t)u(k_i(t))r_{ji}(k - I_i + I_j, t)] + \\
&+ \sum_{\substack{c,s=1 \\ c,s \neq i}}^{n} \mu_s(t)\beta_{sc}(k + I_c - I_s, t)u(k_s(t))\nu_i(k + I_c - I_s, t), i = \overline{1, n}.
\end{aligned} \tag{17}$$

5. Solution of DDE for the closed network with a limited number of spaces of expectations of requests in queueing systems

Consider the closed queueing network with a limited length of the request for the system S_i, equal L_i, $i = \overline{1, n}$. It is assumed that the notification coming in the system S_i to service, takes place in the queue, $i = \overline{1, n}$. If on arrival request into the system S_i the number of requests is less than L_i , then the incoming request takes place in the queue, otherwise it is immediately operated and goes according to the matrix pass $P = \|p_{ij}\|_{n \times n}$ by other system:

$$f_{k_i}^{(i)}(t) = \begin{cases} 1 & \text{if} \quad 0 \leq k_i(t) < L_i, \\ 0 & \text{if} \quad k_i(t) \geq L_i. \end{cases} \tag{18}$$

For the closed network $\psi_{ij}(k, t)$ - conditional probability that the request occupy the i-th system in to a time t , where network is in the state (k, t) immediately after the time of its transfer to the system (Ie, without the transfer of the last report), then will operate in j-th system $\left(\sum_{i=1}^{n} k_i(t) = K - 1, i, j = \overline{1, n}\right)$; $\beta_{ij}(k, t)$ - conditional probability that the request handled in i -th system at the time of t , where network is in the state of (k,t) immediately before the end of its operation, will first serve in j-th queueing system, $\sum_{i=1}^{n} k_i(t) = K, i, j = \overline{1,n}$.

According to the formula for the probability of complete:

$$\psi_{ij}(k,t) = f^{(i)}(k,t)\delta_{ij} + \left(1 - f^{(i)}(k,t)\sum_{i=1}^{n} p_{il}\psi_{lj}(k,t)\right), \quad \sum_{i=1}^{n} k_i(t) = K-1, j = \overline{1,n}, \tag{19}$$

$$\beta_{ij}(k,t) = \sum_{l=1}^{n} p_{il}\psi_{lj}(k - I_i, t), \sum_{i=1}^{n} k_i(t) = K, j = \overline{1,n}. \tag{20}$$

So

$$\sum_{j=1}^{n} \psi_{ij}(k,t) = 1, \quad \sum_{i=1}^{n} k_i(t) = K-1, \tag{21}$$

$$\beta_{ij}(k,t) = 1, \quad \sum_{i=1}^{n} k_i(t) = K. \tag{22}$$

In a closed network must be: if $L_i < K$, $i = \overline{1,n}$, then $\sum_{i=1}^{n} L_i \geq K$ - the number of requests in the network should not be larger than the maximum number of requests that can be accepted by all queueing systems [Gordon-Newell].

Example 1. *Consider a closed network* $n = 3$, $m_i = 1$, $i = \overline{1,3}$, $L_1 = 1$, $L_2 = 2$, $L_3 = 2$, $K = 6$. *Let's assume that* $f^{(i)}(k,t) = f^{(i)}(k_i,t)$. *The number of states in this network is equal 6 which have the form* $\{(0,3,3),$ $(1,2,3), (1,3,2), (2,1,3), (2,2,2), (2,3,1)\}$.

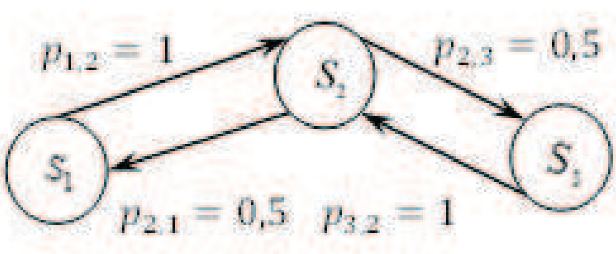

FIGURE 1. Closed queueing network

From the formulas (19), in the case of $k_1(t)+k_2(t)+k_3(t)=K-1$, we have a system of equations:

(23)
$$\begin{cases}\psi_{11}(k,t)=f^{(1)}(k_1,t)+\left(1-f^{(1)}(k_1,t)\right)\psi_{21}(k,t),\\ \psi_{12}(k,t)=\left(1-f^{(1)}(k_1,t)\right)\psi_{22}(k,t),\\ \psi_{13}(k,t)=\left(1-f^{(1)}(k_1,t)\right)\psi_{23}(k,t),\\ \psi_{21}(k,t)=\left(1-f^{(2)}(k_2,t)\right)\left(0,5\psi_{11}(k,t)+0,5\psi_{31}(k,t)\right),\\ \psi_{22}(k,t)=f^{(2)}(k_2,t)+\left(1-f^{(2)}(k_2,t)\right)\left(0,5\psi_{12}(k,t)+0,5\psi_{32}(k,t)\right),\\ \psi_{23}(k,t)=\left(1-f^{(2)}(k_2,t)\right)+\left(0,5\psi_{13}(k,t)+0,5\psi_{33}(k,t)\right),\\ \psi_{31}(k,t)=\left(1-f^{(3)}(k_3,t)\right)\psi_{21}(k,t),\\ \psi_{32}(k,t)=\left(1-f^{(3)}(k_3,t)\right)\psi_{22}(k,t),\\ \psi_{33}(k,t)=f^{(3)}(k_3,t)+\left(1-f^{(3)}(k_3,t)\right)\psi_{23}(k,t).\end{cases}$$

The system of equations (23) with probabilities $\psi_{11}(k,t)$, $\psi_{12}(k,t)$, $\psi_{13}(k,t)$, $\psi_{21}(k,t)$, $\psi_{22}(k,t)$, $\psi_{23}(k,t)$, $\psi_{31}(k,t)$, $\psi_{32}(k,t)$, $\psi_{33}(k,t)$ can be solved using the Mathematica 8 package. Then we get:

(24)
$$\psi_{11}(k,t)=$$
$$=\frac{f^{(1)}(k_1,t)\left(0,5+f^{(2)}(k_2,t)\left(0,5-0,5f^{(3)}(k_3,t)\right)\right)+0,5f^{(3)}(k_3,t)}{0,5f^{(1)}(k_1,t)+f^{(2)}(k_2,t)-0,5\cdot f^{(1)}(k_1,t)f^{(2)}(k_2,t)+0,5f^{(3)}(k_3,t)-0,5f^{(2)}(k_2,t)f^{(3)}(k_3,t)},$$

(25)
$$\psi_{12}(k,t)=$$
$$=\frac{\left(1+f^{(1)}(k_1,t)\right)f^{(2)}(k_2,t)}{0,5f^{(1)}(k_1,t)-f^{(2)}(k_2,t)+0,5f^{(1)}(k_1,t)f^{(2)}(k_2,t)-0,5f^{(3)}(k_3,t)+0,5f^{(2)}(k_2,t)f^{(3)}(k_3,t)},$$

$$\begin{aligned}\psi_{13}(k,t) =& \\ =& \frac{\left(0,5f^{(1)}(k_1,t)\left(0,5-0,5f^{(2)}(k_2,t)\right)+0,5f^{(2)}(k_2,t)\right)f^{(3)}(k_3,t)}{1-0,5f^{(1)}(k_1,t)-f^{(2)}(k_2,t)+0,5f^{(1)}(k_1,t)f^{(2)}(k_2,t)-0,5f^{(3)}(k_3,t)+0,5f^{(2)}(k_2,t)f^{(3)}(k_3,t)}+ \\ &+\frac{\left(1+f^{(1)}(k_1,t)\right)f^{(2)}(k_2,t)}{(0,5f^{(1)}(k_1,t)+f^{(2)}(k_2,t)+0,5f^{(1)}(k_1,t)f^{(2)}(k_2,t)-0,5f^{(3)}(k_3,t)+0,5f^{(2)}(k_2,t)f^{(3)}(k_3,t))}\times \\ &\times\frac{1}{((1-0,5f^{(1)}(k_1,t))+f^{(2)}(k_2,t)+0,5f^{(1)}(k_1,t)f^{(2)}(k_2,t)-0,5f^{(3)}(k_3,t)+0,5f^{(2)}(k_2,t)f^{(3)}(k_3,t))},\end{aligned}\tag{26}$$

$$\begin{aligned}\psi_{21}(k,t) =& \\ &+\frac{\left(1+f^{(1)}(k_1,t)\right)f^{(2)}(k_2,t)}{(0,5f^{(1)}(k_1,t)+f^{(2)}(k_2,t)-0,5f^{(1)}(k_1,t))\,f^{(2)}(k_2,t)+0,5f^{(3)}(k_3,t)-0,5f^{(2)}(k_2,t)f^{(3)}(k_3,t)}\times \\ &\times\frac{1}{(1-f^{(1)}(k_1,t))},\end{aligned}\tag{27}$$

$$\begin{aligned}\psi_{22}(k,t) =& \\ &\frac{f^{(2)}(k_2,t)}{0,5f^{(1)}(k_1,t)+f^{(2)}(k_2,t)-0,5f^{(1)}(k_1,t)f^{(2)}(k_2,t)+0,5f^{(3)}(k_3,t)-0,5f^{(2)}(k_2,t)f^{(3)}(k_3,t)},\end{aligned}\tag{28}$$

$$\begin{aligned}\psi_{23}(k,t) =& \\ &\frac{f^{(2)}(k_2,t)+0,25f^{(1)}(k_1,t)f^{(3)}(k_3,t)+\left(0,5+f^{(1)}(k_1,t)\left(0,5-0,25f^{(2)}(k_2,t)-0,5f^{(2)}(k_2,t)f^{(3)}(k_3,t)\right)\right)}{((0,5f^{(1)}(k_1,t)-f^{(2)}(k_2,t)+0,5f^{(1)}(k_1,t)f^{(2)}(k_2,t)-f^{(3)}(k_3,t)+0,5f^{(2)}(k_2,t)f^{(3)}(k_3,t))}\times \\ &\times\frac{1}{(1-0,5f^{(1)}(k_1,t)-f^{(2)}(k_2,t)+0,5f^{(1)}(k_1,t)f^{(2)}(k_2,t)-0,5f^{(3)}(k_3,t)+0,5f^{(2)}(k_2,t)f^{(3)}(k_3,t))},\end{aligned}\tag{29}$$

$$\begin{aligned}\psi_{31}(k,t) =& \\ &\frac{0,5f^{(1)}(k_1,t)\left(1+f^{(2)}(k_2,t)\left(1-f^{(3)}(k_3,t)\right)+f^{(1)}(k_1,t)\left(1-f^{(2)}(k_2,t)\left(1-f^{(3)}(k_3,t)\right)+f^{(3)}(k_3,t)\right)+f^{(3)}(k_3,t)\right)}{(1-0,5f^{(1)}(k_1,t))\,(0,5f^{(1)}(k_1,t)+f^{(2)}(k_2,t)-0,5f^{(1)}(k_1,t)f^{(2)}(k_2,t)+0,5f^{(3)}(k_3,t)-0,5f^{(2)}(k_2,t)f^{(3)}(k_3,t))},\end{aligned}\tag{30}$$

(31)
$$\psi_{32}(k,t) = \frac{f^{(2)}(k_2,t)\left(1-f^{(3)}(k_3,t)\right)}{0,5f^{(1)}(k_1,t)-f^{(2)}(k_2,t)+0,5f^{(1)}(k_1,t)f^{(2)}(k_2,t)-0,5f^{(3)}(k_3,t)+0,5f^{(2)}(k_2,t)f^{(3)}(k_3,t)},$$

(32)
$$\psi_{33}(k,t) = \frac{0,5\left(1+f^{(1)}(k_1,t)\left(1-f^{(2)}(k_2,t)\right)-0,5f^{(2)}(k_2,t)\right)f^{(3)}(k_3,t)}{1-0,5f^{(1)}(k_1,t)-f^{(2)}(k_2,t)+0,5f^{(1)}(k_1,t)f^{(2)}(k_2,t)-0,5f^{(3)}(k_3,t)+0,5f^{(2)}(k_2,t)f^{(3)}(k_3,t)}.$$

For queueing network, according to formulas (24)-(32), conditional probabilities $\psi_{ij}(k,t)$ take the form:

$$\begin{array}{ll}
\psi_{11}(1,1,3,t)=0, & \psi_{31}(1,1,3,t)=0,\\
\psi_{11}(2,0,3,t)=0, & \psi_{31}(2,0,3,t)=0,\\
\psi_{11}(0,2,3,t)=0,25, & \psi_{31}(0,2,3,t)=0,125,\\
\psi_{11}(2,1,2,t)=0, & \psi_{31}(2,1,2,t)=0,\\
\psi_{11}(1,2,2,t)=0,25, & \psi_{31}(1,2,2,t)=0,\\
\psi_{13}(1,1,3,t)=0,75, & \psi_{33}(1,1,3,t)=0,\\
\psi_{13}(2,0,3,t)=0,5, & \psi_{33}(2,0,3,t)=0,\\
\psi_{13}(2,1,2,t)=0,5, & \psi_{33}(2,1,2,t)=0,125,\\
\psi_{13}(2,2,1,t)=0, & \psi_{33}(2,2,1,t)=0,125,\\
\psi_{22}(0,2,3,t)=0, & \psi_{22}(0,3,2,t)=0,\\
\psi_{23}(1,2,2,t)=0, & \psi_{23}(1,3,1,t)=0,\\
\psi_{21}(2,2,1,t)=0, & \psi_{22}(2,3,0,t)=0.
\end{array}$$

If $k_1(t)+k_2(t)+k_3(t)=K$, then from pattern (20) we will have:

$$\beta_{11}(k,t)=\psi_{21}(k-I_1,t)=$$
$$=\frac{f^{(1)}(k_1-1,t)\left(0,5+f^{(1)}(k_1-1,t)\left(0,5-0,5f^{(2)}(k_2,t)\right)\right)-0,5f^{(2)}(k_2,t)}{\left(1-f^{(1)}(k_1-1,t)\right)\left(0,5f^{(1)}(k_1-1,t)+f^{(2)}(k_2,t)-0,5f^{(1)}(k_1-1,t)\right)f^{(2)}(k_2,t)+f^{(3)}(k_3,t)-0,5f^{(2)}(k_2,t)f^{(3)}(k_3,t)},$$

$$\beta_{12}(k,t)=\psi_{22}(k-I_1,t)=$$
$$=\frac{f^{(2)}(k_2,t)}{0,5f^{(1)}(k_1-1,t)+f^{(2)}(k_2,t)-0,5f^{(1)}(k_1-1,t)f^{(2)}(k_2,t)+0,5f^{(3)}(k_3,t)-0,5f^{(2)}(k_2,t)f^{(3)}(k_3,t)},$$

$$\beta_{13}(k,t)=\psi_{23}(k-I_1,t)=$$
$$=\frac{f^{(2)}(k_2,t)+0,25f^{(1)}(k_1-1,t)f^{(3)}(k_3,t)+0,5+f^{(1)}(k_1-1,t)\left(0,5-0,25f^{(2)}(k_2,t)-0,5f^{(2)}(k_2,t)f^{(3)}(k_3,t)\right)}{\left(0,5f^{(1)}(k_1-1,t)-f^{(2)}(k_2,t)+0,5f^{(1)}(k_1-1,t)f^{(2)}(k_2,t)-f^{(3)}(k_3,t)+0,5f^{(2)}(k_2,t)f^{(3)}(k_3,t)\right)}\times$$
$$\times\frac{1}{\left(1-0,5f^{(1)}(k_1-1,t)-f^{(2)}(k_2,t)+0,5f^{(1)}(k_1-1,t)f^{(2)}(k_2,t)-0,5f^{(3)}(k_3,t)+0,5f^{(2)}(k_2,t)f^{(3)}(k_3,t)\right)},$$

$$\beta_{31}(k,t)=\psi_{21}(k-I_3,t)=$$
$$=\frac{0,5\left(f^{(1)}(k_1-1,t)\left(1+f^{(1)}(k_1,t)\left(1-f^{(2)}(k_2,t)\right)\right)-f^{(2)}(k_2,t)\right)}{\left(1-f^{(1)}(k_1,t)\right)\left(0,5f^{(1)}(k_1-1,t)+f^{(2)}(k_2,t)-0,5f^{(1)}(k_1,t)\right)f^{(2)}(k_2,t)+0,5f^{(3)}(k_3-1,t)-0,5f^{(2)}(k_2,t)f^{(3)}(k_3-1,t)},$$

$$\beta_{32}(k,t)=\psi_{22}(k-I_3,t)=$$
$$=\frac{f^{(2)}(k_2,t)}{0,5f^{(1)}(k_1,t)+f^{(2)}(k_2,t)-0,5f^{(1)}(k_1,t)f^{(2)}(k_2,t)+0,5f^{(3)}(k_3-1,t)-0,5f^{(2)}(k_2,t)f^{(3)}(k_3-1,t)},$$

$$\beta_{33}(k,t)=\psi_{23}(k-I_3,t)=$$
$$=\frac{f^{(2)}(k_2,t)+0,25f^{(1)}(k_1,t)f^{(3)}(k_3-1,t)+0,5+f^{(1)}(k_1,t)\left(0,5-0,25f^{(2)}(k_2,t)-0,5f^{(2)}(k_2,t)f^{(3)}(k_3-1,t)\right)}{\left(0,5f^{(1)}(k_1,t)-f^{(2)}(k_2,t)+0,5f^{(1)}(k_1,t)f^{(2)}(k_2,t)-0,5f^{(3)}(k_3-1,t)+0,5f^{(2)}(k_2,t)f^{(3)}(k_3-1,t)\right)}\times$$
$$\times\frac{1}{\left(1-0,5f^{(1)}(k_1,t)-f^{(2)}(k_2,t)+0,5f^{(1)}(k_1,t)f^{(2)}(k_2,t)-0,5f^{(3)}(k_3-1,t)+0,5f^{(2)}(k_2,t)f^{(3)}(k_3-1,t)\right)},$$

$$\beta_{21}(k,t) = 0,5\psi_{11}(k-I_2,t) + 0,5\psi_{31}(k-I_2,t) =$$

$$= 0,5\left(\frac{f^{(1)}(k_1,t)\left(0,5+f^{(2)}(k_2-1,t)\left(0,5-0,5f^{(3)}(k_3,t)\right)\right)+0,5f^{(3)}(k_3,t)}{0,5f^{(1)}(k_1,t)+f^{(2)}(k_2,t)-0,5f^{(1)}(k_1,t)f^{(2)}(k_2,t)+0,5f^{(3)}(k_3-1,t)-0,5f^{(2)}(k_2-1,t)f^{(3)}(k_3,t)}\right)+$$

$$+0,5\left(\frac{0,5f^{(1)}(k_1,t)\left(1+f^{(2)}(k_2-1,t)\left(1-f^{(3)}(k_3,t)\right)+f^{(1)}(k_1,t)\left(1-f^{(2)}(k_2-1,t)\left(1-f^{(3)}(k_3,t)\right)+f^{(3)}(k_3,t)\right)+f^{(3)}(k_3,t)\right)}{\left(1-f^{(1)}(k_1,t)\right)\left(0,5f^{(1)}(k_1,t)+f^{(2)}(k_2-1,t)-0,5f^{(1)}(k_1,t)f^{(2)}(k_2-1,t)+0,5f^{(3)}(k_3,t)-0,5f^{(2)}(k_2-1,t)\right)f^{(3)}(k_3,t)}\right),$$

$$\beta_{22}(k,t) = 0,5\psi_{12}(k-I_2,t) + 0,5\psi_{32}(k-I_2,t) =$$

$$= 0,5\left(\frac{\left(1+f^{(1)}(k_1,t)\right)f^{(2)}(k_2-1,t)}{0,5f^{(1)}(k_1,t)-f^{(2)}(k_2-1,t)+0,5f^{(1)}(k_1,t)f^{(2)}(k_2-1,t)-0,5f^{(3)}(k_3,t)+0,5f^{(2)}(k_2-1,t)f^{(3)}(k_3,t)}\right)+$$
$$+\left(\frac{f^{(2)}(k_2-1,t)\left(1-f^{(3)}(k_3,t)\right)}{0,5f^{(1)}(k_1,t)-f^{(2)}(k_2-1,t)+0,5f^{(1)}(k_1,t)f^{(2)}(k_2-1,t)-0,5f^{(3)}(k_3,t)+0,5f^{(2)}(k_2-1,t)f^{(3)}(k_3,t)}\right),$$

$$\beta_{23}(k,t) = 0,5\psi_{13}(k-I_2,t) + 0,5\psi_{33}(k-I_2,t) =$$

$$= 0,5\left(\frac{\left(0,5+f^{(1)}(k_1,t)\left(0,5-0,5f^{(2)}(k_2-1,t)\right)+0,5f^{(2)}(k_2-1,t)\right)f^{(3)}(k_3,t)}{1-0,5f^{(1)}(k_1,t)-f^{(2)}(k_2-1,t)+0,5f^{(1)}(k_1,t)f^{(2)}(k_2-1,t)-0,5f^{(3)}(k_3,t)+0,5f^{(2)}(k_2-1,t)f^{(3)}(k_3,t)}\right)+$$
$$+\frac{\left(1+f^{(1)}(k_1,t)\right)f^{(2)}(k_2-1,t)}{\left(0,5f^{(1)}(k_1,t)+f^{(2)}(k_2-1,t)+0,5f^{(1)}(k_1,t)f^{(2)}(k_2-1,t)-0,5f^{(3)}(k_3,t)+0,5f^{(2)}(k_2-1,t)f^{(3)}(k_3,t)\right)}\times$$
$$\times\frac{1}{\left(\left(1-f^{(1)}(k_1,t)\right)+f^{(2)}(k_2-1,t)+0,5\left(f^{(1)}(k_1,t)f^{(2)}(k_2-1,t)-\left(f^{(3)}(k_3,t)-f^{(2)}(k_2-1,t)f^{(2)}(k_3,t)\right)\right)\right)}+$$
$$+0,5\left(\frac{0,5\left(1+f^{(1)}(k_1,t)\left(1-f^{(2)}(k_2-1,t)\right)-0,5f^{(2)}(k_2-1,t)\right)f^{(3)}(k_3,t)}{1-0,5f^{(1)}(k_1,t)-f^{(2)}(k_2-1,t)+0,5f^{(1)}(k_1,t)f^{(2)}(k_2-1,t)-0,5f^{(3)}(k_3,t)+0,5f^{(2)}(k_2-1,t)f^{(3)}(k_3,t)}\right).$$

Considering the above, you can find conditional probability values $\beta_{ij}(k,t)$ *for each service system for each network state* $k, i, j = \overline{1,3}$*. By definition* $\beta_{ij}(k,t)$ *shows that* $\beta_{ij}(k,t) = 0$*,* $i = \overline{1,3}$*. In addition, some transitions between network states are impossible, which is due to the probability matrix between the states of the network. Therefore, the corresponding conditional probabilities* $\beta_{ij}(k,t)$ *are also equal to zero. We will write conditional probabilities* $\beta_{ij}(k,t)$*,* $i = \overline{1,3}$*:*

$$\beta_{21}(1,2,3,t) = 0,5\psi_{11}(1,1,3,t) + 0,5\psi_{31}(1,1,3,t) = 0 + 0 = 0,$$
$$\beta_{21}(2,1,3,t) = 0,5\psi_{11}(2,0,3,t) + 0,5\psi_{31}(2,0,3,t) = 0 + 0 = 0,$$
$$\beta_{21}(0,3,3,t) = 0,5\psi_{11}(0,2,3,t) + 0,5\psi_{31}(0,2,3,t) = 0,25 + 0,125 = 0,375,$$
$$\beta_{21}(2,2,2,t) = 0,5\psi_{11}(2,1,2,t) + 0,5\psi_{31}(2,1,2,t) = 0 + 0 = 0,$$
$$\beta_{21}(1,3,2,t) = 0,5\psi_{11}(1,2,2,t) + 0,5\psi_{31}(1,2,2,t) = 0,25 + 0 = 0,25,$$
$$\beta_{23}(1,2,3,t) = 0,5\psi_{13}(1,1,3,t) + 0,5\psi_{33}(1,1,3,t) = 0,75 + 0 = 0,75,$$
$$\beta_{23}(2,1,3,t) = 0,5\psi_{13}(2,0,3,t) + 0,5\psi_{33}(2,0,3,t) = 0,5 + 0 = 0,5,$$
$$\beta_{23}(2,2,2,t) = 0,5\psi_{13}(2,1,2,t) + 0,5\psi_{33}(2,1,2,t) = 0,5 + 0,125 = 0,625,$$
$$\beta_{23}(2,3,1,t) = 0,5\psi_{13}(2,2,1,t) + 0,5\psi_{31}(2,1,2,t) = 0 + 0,125 = 0,125,$$
$$\beta_{12}(0,3,3,t) = 0,$$
$$\beta_{12}(1,2,3,t) = \psi_{22}(0,2,3,t) = 0,$$
$$\beta_{12}(1,3,2,t) = \psi_{22}(0,3,2,t) = 0,$$
$$\beta_{13}(2,2,2,t) = \psi_{23}(1,2,2,t) = 0,$$
$$\beta_{13}(2,3,1,t) = \psi_{23}(1,3,1,t) = 0,$$
$$\beta_{31}(2,2,2,t) = \psi_{21}(2,2,1,t) = 0,$$
$$\beta_{32}(2,3,1,t) = \psi_{22}(2,3,0,t) = 0.$$

Let $r_i(k) = 4$ *bytes be the volume of one request. Let* $r_{ij}(k,t)$ *- linear because of* t *and independent of state of* k *equals* $20t$ *bytes. The intensity of handling requests in the queueing network system are equal* $\mu_1(t) = \cos(4t) + 1$*,* $\mu_2(t) = 5\cos(4t) + 5$*,* $\mu_3(t) = 5\cos(4t) + 1$*. Let's take the initial conditions:* $\nu_1(0,3,3,t) = 0$*,* $\nu_1(1,2,3,t) = 20$*,* $\nu_1(1,3,2,t) = 20$*,* $\nu_1(2,1,3,t) = 40$*,* $\nu_1(2,2,2,t) = 40$*,* $\nu_1(2,3,1,t) = 40$*,* $\nu_2(0,3,3,t) = 60$*,* $\nu_2(1,2,3,t) = 40$*,* $\nu_2(1,3,2,t) = 60$*,* $\nu_2(2,1,3,t) = 10$*,* $\nu_2(2,2,2,t) = 40$*,* $\nu_2(2,3,1,t) = 60$*,* $\nu_3(0,3,3,t) = 60$*,* $\nu_3(1,2,3,t) = 60$*,* $\nu_3(1,3,2,t) = 40$*,* $\nu_3(2,1,3,t) = 60$*,* $\nu_3(2,2,2,t) = 40$*,* $\nu_3(2,3,1,t) = 20$*.*
Then the system of DDE (17) takes the form:

$$\frac{\mathrm{d}\nu_1(0,3,3,t)}{\mathrm{d}t} = -7\left(1 + \cos(4t)\right)\nu_1(0,3,3,t) + 4,$$
$$\frac{\mathrm{d}\nu_1(1,2,3,t)}{\mathrm{d}t} = -11\left(1 + \cos(4t)\right)\nu_1(1,2,3,t) + 4,$$
$$\frac{\mathrm{d}\nu_1(1,3,2,t)}{\mathrm{d}t} = -11\left(1 + \cos(4t)\right)\nu_1(1,3,2,t) + 4,$$
$$\frac{\mathrm{d}\nu_1(2,1,3,t)}{\mathrm{d}t} = -11\left(1 + \cos(4t)\right)\nu_1(2,1,3,t) + 4,$$
$$\frac{\mathrm{d}\nu_1(2,2,2,t)}{\mathrm{d}t} = -11\left(1 + \cos(4t)\right)\nu_1(2,2,2,t) + 4 - 5t\left(1 + \cos(4t)\right),$$

$$\frac{d\nu_1(2,3,1,t)}{dt} = -11\,(1+\cos(4t))\,\nu_1(2,3,1,t) + 4,$$

$$\frac{d\nu_2(0,3,3,t)}{dt} = -7\,(1+\cos(4t))\,\nu_2(0,3,3,t) + 4,$$

$$\frac{d\nu_2(1,2,3,t)}{dt} = -7\,(1+\cos(4t))\,\nu_2(1,2,3,t) + 4,$$

$$\frac{d\nu_2(1,3,2,t)}{dt} = -7\,(1+\cos(4t))\,\nu_2(1,3,3,t) + 3,75\,(1+\cos(4t))\,\nu_2(1,3,3,t) + 4 - 75t\,(1+\cos(4t))\,,$$

$$\frac{d\nu_2(2,1,3,t)}{dt} = -7\,(1+\cos(4t))\,\nu_2(2,1,3,t) + 4,$$

$$\frac{d\nu_2(2,2,2,t)}{dt} = -7\,(1+\cos(4t))\,\nu_2(2,2,2,t) + 5\,(1+\cos(4t))\,\nu_2(2,1,3,t) + 4 - 50t\,(1+\cos(4t))\,,$$

$$\frac{d\nu_2(2,3,1,t)}{dt} = -7\,(1+\cos(4t))\,\nu_2(2,3,1,t) + 3,125\,(1+\cos(4t))\,\nu_2(2,2,2,t) + 4 - 12,5t,$$

$$\frac{d\nu_3(0,3,3,t)}{dt} = -7\,(1+\cos(4t))\,\nu_3(0,3,3,t) + 4,$$

$$\frac{d\nu_2(1,2,3,t)}{dt} = -7\,(1+\cos(4t))\,\nu_2(1,2,3,t) + 4,$$

$$\frac{d\nu_3(0,3,3,t)}{dt} = -7\,(1+\cos(4t))\,\nu_2(0,3,3,t) + 4,$$

$$\frac{d\nu_3(1,2,3,t)}{dt} = -7\,(1+\cos(4t))\,\nu_2(1,2,3,t) + 4,$$

$$\frac{d\nu_3(1,3,2,t)}{dt} = -7\,(1+\cos(4t))\,\nu_3(1,3,2,t) + 7,5\,(1+\cos(4t))\,\nu_3(1,2,3,t) + 75t\,(1+\cos(4t)) + 4,$$

$$\frac{d\nu_3(2,1,3,t)}{dt} = -7\,(1+\cos(4t))\,\nu_3(2,1,3,t) + 4,$$

$$\frac{d\nu_3(2,2,2,t)}{dt} = -7\,(1+\cos(4t))\,\nu_3(2,2,2,t) + 7,5\,(1+\cos(4t))\,\nu_3(2,3,1,t) + 75t\,(1+\cos(4t)) + 4,$$

$$\frac{d\nu_3(2,3,1,t)}{dt} = -7\,(1+\cos(4t))\,\nu_3(2,3,1,t) + 6,25\,(1+\cos(4t))\,\nu_3(2,2,2,t) + 60t\,(1+\cos(4t)) + 4.$$

This system can be solved using the Mathematica 8 package and get a numerical solution. This solution is shown in the form of expected volumes of requests charts for each queueing system of network in the figures 1-4.

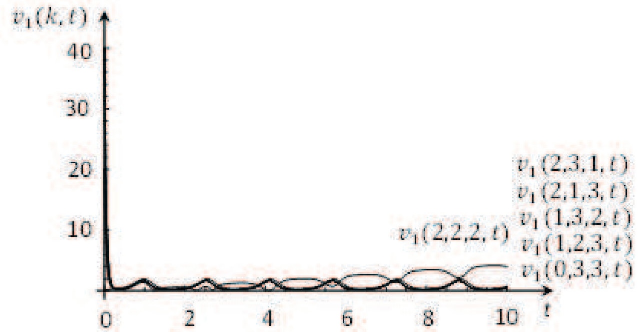

FIGURE 2. The volumes of requests in the system S_1 for $T = [0, 10]$

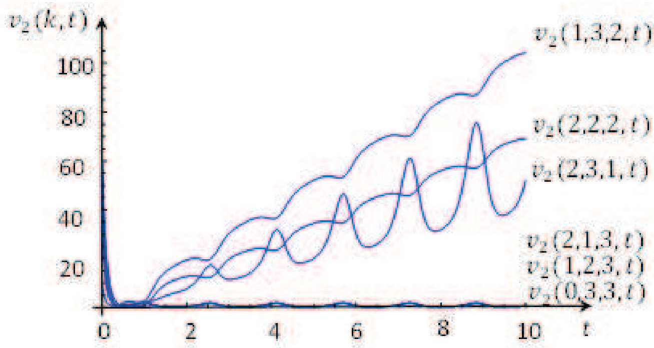

FIGURE 3. The volumes of requests in the system S_2 for $T = [0, 10]$

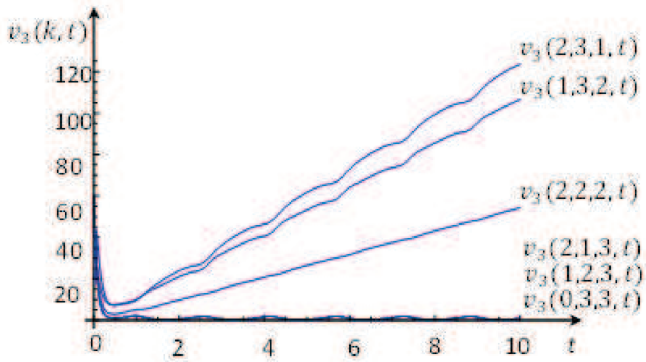

FIGURE 4. The volumes of requests in the system S_3 for $T = [0, 10]$

We can also find analytical solutions for the expected volumes of requests in queueing network systems. Then we get:

$$\nu_1(0,3,3,t) = 4\mathrm{e}^{\int_0^t 7(\cos(4x)+1)\mathrm{d}x} \int_0^t \mathrm{e}^{-\int_0^x 7(\cos(4y)+1)\mathrm{d}y}\mathrm{d}x,$$

$$\nu_1(1,2,3,t) = 4\mathrm{e}^{\int_0^t 11(\cos(4x)+1)\mathrm{d}x} \left[5 + \int_0^t \mathrm{e}^{-\int_0^x 11(\cos(4y)+1)\mathrm{d}y}\mathrm{d}x\right],$$

$$\nu_1(1,3,2,t) = 4\mathrm{e}^{\int_0^t 11(\cos(4x)+1)\mathrm{d}x} \left[5 + \int_0^t \mathrm{e}^{-\int_0^x 11(\cos(4y)+1)\mathrm{d}y}\mathrm{d}x\right],$$

$$\nu_1(2,1,3,t) = 4\mathrm{e}^{\int_0^t 11(\cos(4x)+1)\mathrm{d}x} \left[10 + \int_0^t \mathrm{e}^{-\int_0^x 11(\cos(4y)+1)\mathrm{d}y}\mathrm{d}x\right],$$

$$\nu_1(2,2,2,t) = \mathrm{e}^{\int_0^t 11(\cos(4x)+1)\mathrm{d}x} \left[40 + \int_0^t (4 - 5t\,(1+\cos(4t)))\,\mathrm{e}^{-\int_0^x 11(\cos(4y)+1)\mathrm{d}y}\mathrm{d}x\right],$$

$$\nu_1(2,3,1,t) = 4\mathrm{e}^{\int_0^t 11(\cos(4x)+1)\mathrm{d}x} \left[10 + \int_0^t \mathrm{e}^{-\int_0^x 11(\cos(4y)+1)\mathrm{d}y}\mathrm{d}x\right],$$

$$\nu_1(0,3,3,t) = 4\mathrm{e}^{\int_0^t 7(\cos(4x)+1)\mathrm{d}x} \left[15 + \int_0^t \mathrm{e}^{-\int_0^x 7(\cos(4y)+1)\mathrm{d}y}\mathrm{d}x\right],$$

$$\nu_1(1,2,3,t) = 4\mathrm{e}^{\int_0^t 7(\cos(4x)+1)\mathrm{d}x}\left[10+\int_0^t \mathrm{e}^{-\int_0^x 7(\cos(4y)+1)\mathrm{d}y}\mathrm{d}x\right],$$

$$\nu_1(1,3,2,t) = \mathrm{e}^{\int_0^t 7(\cos(4x)+1)\mathrm{d}x}\Bigg[60+\int_0^t \Bigg(4-75x\,(1+\cos(4x))+$$

$$+3,75\,(1+\cos(4x))\left(4\mathrm{e}^{\int_0^x 7(\cos(4z)+1)\mathrm{d}z}\left[15+\int_0^x \mathrm{e}^{-\int_0^s 7(\cos(4l)+1)\mathrm{d}l}\mathrm{d}s\right]\right)\Bigg)\mathrm{e}^{-\int_0^x 7(\cos(4y)+1)\mathrm{d}y}\mathrm{d}x\Bigg],$$

$$\nu_2(2,1,3,t) = 2\mathrm{e}^{\int_0^t 7(\cos(4x)+1)\mathrm{d}x}\left[5+2\int_0^t \mathrm{e}^{-\int_0^x 7(\cos(4y)+1)\mathrm{d}y}\mathrm{d}x\right]$$

$$\nu_2(2,2,2,t) = \mathrm{e}^{\int_0^t 7(\cos(4x)+1)\mathrm{d}x}\Bigg[40+\int_0^t \Bigg(4-50x\,(1+\cos(4x))+$$

$$+5\,(1+\cos(4x))\left(2\mathrm{e}^{\int_0^x 7(\cos(4z)+1)\mathrm{d}z}\left[5+2\int_0^x \mathrm{e}^{-\int_0^s 7(cos(4l)+1)\mathrm{d}l}\mathrm{d}s\right]\right)\Bigg)\mathrm{e}^{-\int_0^x 7(cos(4y)+1)\mathrm{d}y}\mathrm{d}x\Bigg],$$

$$\nu_2(2,3,1,t) = \mathrm{e}^{\int_0^t 7(\cos(4x)+1)\mathrm{d}x}\Bigg[60+\int_0^t \Bigg(4-12,5x+3,125\,(1+\cos(4x))\times$$

$$\times\mathrm{e}^{\int_0^x 7(\cos(4l)+1)\mathrm{d}l}\Bigg[60+\int_0^x \Bigg(4-50h\,(1+\cos(4h))+5\,(1+\cos(4h))\times$$

$$\times\left(4\mathrm{e}^{\int_0^h 7(\cos(4k)+1)\mathrm{d}k}\int_0^h \mathrm{e}^{-\int_0^s 7(\cos(4w)+1)\mathrm{d}w}\mathrm{d}s\right)\Bigg)\mathrm{e}^{-\int_0^h 7(\cos(4y)+1)\mathrm{d}y}\mathrm{d}h\Bigg)\Bigg]\Bigg)\mathrm{e}^{-\int_0^x 7(\cos(4y)+1)\mathrm{d}y}\mathrm{d}x,$$

$$\nu_3(0,3,3,t) = 4\mathrm{e}^{\int_0^t 7(\cos(4x)+1)\mathrm{d}x}\left[15+\int_0^t \mathrm{e}^{-\int_0^x 7(\cos(4y)+1)\mathrm{d}y}\mathrm{d}x\right],$$

$$\nu_3(1,2,3,t) = 4\mathrm{e}^{\int_0^t 7(\cos(4x)+1)\mathrm{d}x}\left[15+\int_0^t \mathrm{e}^{-\int_0^x 7(\cos(4y)+1)\mathrm{d}y}\mathrm{d}x\right],$$

$$\nu_3(1,3,2,t) = 4\mathrm{e}^{\int_0^t 7(\cos(4x)+1)\mathrm{d}x}\Bigg[10+\int_0^t \Bigg(4+75x\,(1+\cos(4x))+7,5x\,(1+\cos(4x))\times$$

$$\times\left(4\mathrm{e}^{\int_0^x 7(\cos(4z)+1)\mathrm{d}z}\left[15+\int_0^t \mathrm{e}^{-\int_0^q 7(\cos(4y)+1)\mathrm{d}y}\mathrm{d}q\right]\right)\Bigg)\mathrm{e}^{-\int_0^x 7(\cos(4y)+1)\mathrm{d}y}\mathrm{d}x\Bigg],$$

$$\nu_3(2,1,3,t) = 4\mathrm{e}^{\int_0^t 7(\cos(4x)+1)\mathrm{d}x}\left[15+\int_0^t \mathrm{e}^{-\int_0^x 7(\cos(4y)+1)\mathrm{d}y}\mathrm{d}x\right],$$

$$\nu_3(2,2,2,t) = 4\mathrm{e}^{\int_0^t 7(\cos(4x)+1)\mathrm{d}x}\Bigg[10+\int_0^t \Bigg(4+12,5x\,(1+\cos(4x))+1,25\,(1+\cos(4x))\times$$

$$\times\left(\nu_3(2,3,1,t)\right)\mathrm{e}^{-\int_0^x 7(\cos(4y)+1)\mathrm{d}y}\mathrm{d}x\Bigg]$$

$$\nu_3(2,2,2,t) = 4\mathrm{e}^{\int_0^t 7(\cos(4x)+1)\mathrm{d}x}\Bigg[5+\int_0^t \Bigg(4+62,5x\,(1+\cos(4x))+7,25\,(1+\cos(4x))\times$$

$$\times\left(\nu_3(3,2,2,t)\right)\mathrm{e}^{-\int_0^x 7(\cos(4y)+1)\mathrm{d}y}\mathrm{d}x\Bigg].$$

6. Conclusions

We present a method of finding the expected volume of requests in open HM-network with homogeneous requests, bypass of queueing network systems of requests. Were considered a case where the changes in volumes associated with transitions between states of the network are deterministic functions dependent states of network and time, and service systems are single line, assuming that the probability of network systems states, the parameters of entrance flow of messages and service depend on time.

These results can be used to find the amount of memory in information systems. A model of wireless computer network operation with limited number of concurrent connections is presented. This network was analyzed with homogeneous requests, bypass of service nodes, and time-dependent parameters of stream of requests. The conditional probabilities for the service of requests are given by (1)-(6).

A set of differential equations was obtained for the expected volume of homogeneous requests in network systems with limited number of waiting places and an example of a solution of the equation system for this network is shown in Figure 1, where the probability of joining the request to queues in queueing systems depends only on the number of requests in them. Graphs the expected volume of requests for queueing network systems can be found in Figures 2-4.

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