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MAGNETIC INTERLAYER COUPLING ACROSS PARABOLIC QUANTUM-WELL

W. Gruhn

Institute of Physics, JD University, Armii Krajowej Ave 13/15, 42-201 Częstochowa, Poland
e-mail: w.gruhn@ajd.czest.pl

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ABSTRACT

A qualitatively different mechanism of magnetic oscillations within non-square quantum wells has been found compared to the conventional magnetic superlattices which are formed of quantum wells with rectangular potential profiles. In our contribution we will present theory of interlayer coupling in the case of parabolic quantum-well system. Contrary to the conventional models based on perturbative or total energy approaches we exploit the similarities of the quasi-2D electron gas oscillations within parabolic quantum well to the de Haas-van Alphen effect. We derive formula for the interlayer coupling parameters as the function of potential barrier heights and the nonmagnetic layer thickness. Applications of the results obtained to the description of real systems will be widely discussed.

INTRODUCTION

The semimagnetic semiconductors form a model magnetic system with metallic electrical properties. In view of fundamental and technological aspects the problem of nonuniform doping is becoming very important. Modern MBE techniques allow fabrication of semiconductor structures with highly controlled variable chemical composition and thickness of layers. Non-rectangular quantum wells involving substantially nonhomogenous parts, rather than having the conventional rectangular profiles, have been studied both experimentally and theoretically in the past few years and found to exhibit attractive performances (see [1,2], and references therein). The most promising appear the parabolic quantum well (PQW) systems, which have the ability to absorb light only at the bare harmonic-oscillator frequency irrespective of the electron-electron interaction or the number of electrons in the well. From this reason the magnetic PQW systems are interesting as a building blocks spintronic devices. Potentially the most promising are the PQW magnetic superlattices, which consist of alternating magnetic layers and PQW systems. The aim of the paper is to give analytical description of magnetic properties of a PQW superlattice.

High concentration of the free charge carriers causes the indirect spin-spin exchange interaction to be responsible for their magnetic properties [3]. Conventional calculation of the indirect exchange integrals assumes uniform density of the free charge carriers, that mediate the RKKY interaction. However, in the case of PQW structure the confinement of the mobile electrons within resulting quantum well makes this assumption doesn't hold anymore. That's why there is only limited progress in description of magnetic interactions in then on

square-QW superlattices [4,5,6,7]. Generally, the magnetic properties of metallic-like system are determined by the density of mobile charge carriers that mediate this interaction and their spectra. This is why the magnetism of electron system confined within non-square quantum-well must be considered independently. In our contribution we will present theory of the oscillatory interlayer coupling in the case of PQW system. Contrary to the conventional models based on perturbative or total energy approaches we exploit the similarities of the quasi-2DEG oscillations within PQW to the de Haas-van Alphen effect [8,9]. We prove that the effective spin polarization of the electrons/holes that are confined within PQW shows oscillatory behaviour as the function of the PQW (spacer layer) layer thickness.

THE PARABOLIC QUANTUM WELL SYSTEM

Semimagnetic semiconductors are II-VI, IV-VI or III-VI compounds, in which a fraction of nonmagnetic cations (up to the solubility limit) has been substituted by TM and/or RE metal ions [10]. Due to the doping the semimagnetic semiconductors form a model diluted magnetic system with metallic electrical properties. Indirect magnetic interaction in these systems is not different in principle from that in more familiar metallic ferromagnets, yet the size of the effect differs by at least an order of magnitude. Nevertheless, provided that the concentration of the free carriers p exceeds critical $p > p_c \approx 3 \cdot 10^{20}\ \mathrm{cm}^{-3}$ the indirect interaction (RKKY) in a semimagnetic semiconductor system dominates the other mechanisms of magnetic interionic coupling [11]. Usually, it is assumed that magnetic dopants are distributed at random, however, the MBE structures with non-random distribution can be easily obtained [12]. The special case of non-random distribution are the semiconducting magnetic superlattices [13,14]. In these systems the mobile charge carriers are confined within quantum-wells, thus the potential barriers impose constraints onto their motion. As a result of this the (uniform) free electron density and/or energy spectrum are perturbed. Let us consider the energy spectrum in the PQW structure formed of semimagnetic semiconductors. Assuming that the "z" denotes the direction of the planar PQW, the Hamiltonian that describes the electronic structure within the envelope function formalism and effective mass approximation, reads [1].

$$H_\alpha = -\frac{\hbar^2}{2m_\alpha}\nabla^2 + \frac{K_\alpha}{2}z^2 + E_\alpha . \qquad (1)$$

Here, α represents the band index and denotes the electron or hole band, while E_α and m_α the band gap energy and the effective mass at the center of the PQW, respectively. K_α is the curvature of the parabolic potential profile, which is assumed to be infinitely high. The electronic (hole) states within the PQW are those of the standard harmonic oscillator. Let us focus our attention on the single band model, then we can write the electron/hole spectrum as [1].

$$\varepsilon_k = -\frac{\hbar^2 k^2}{2m} + \hbar\omega\left(n+\frac{1}{2}\right) - \mu, \qquad (2)$$

where μ is the Fermi energy. Provided the external magnetic field is applied then to eigenenspectrum (2) the term $g\sigma\mu_B B$ should be added, where g is the electron (hole) g-factor, μ_B is the Bohr magneton while the spin σ we assume to take values $\sigma = \pm 1/2$. As we can see from Eq. (2) the finite thickness and broken translational symmetry of PQW structure leads to different quantization in the growth direction of the allowed electronic states. Perturbational calculation to the RKKY interactions involves summation (integration) over dynamical states of the mobile charge carriers. Therefore the free electron density plays a key role in determination of the spatial dependence of the RKKY exchange integrals. Perturbative calculations of the interspin interactions lead to an expression for the RKKY exchange integral in the PQW system (for details see [7,15])

$$\chi(r) \approx \frac{\chi_0}{r^d} r^2 \left[J_{d/2-1}(x) Y_{d/2-1}(x) + J_{d/2}(x) Y_{d/2}(x) \right], \qquad (3)$$

with $Y_\nu(x)$ being the Neumann function [7] and d = 4 is the effective spectral dimension of the PQW system. It is worthwhile to mention here that in general in the case quasi-2D electron gas systems the effective spectral dimension can take any value from the $1 \le d \le 4$ range [16]. Expression (3) is valid only when the interacting spins are displaced in the lateral direction. However, from both basic as well as applicational point of view the interaction across the PQW layer is more interesting. To gain information about it we should apply other approach [17,18].

OSCILLATIONS OF THERMODYNAMICAL POTENTIAL WITHIN PQW

From Eq. (2) we can see that mobile electrons/hole energy eigenstate can be labeled by $\left(k_x k_y, \omega_D\right)$ where $k \in \mathbf{R}^2$ labels kinetic energy associated with the motion in the lateral direction while ω_D labels energy states generated by the in-layer confinement. Characteristic feature of all systems that exhibit this type of spectrum are the electron density oscillations when an oscillator level crosses the Fermi energy. In conventional dHvA effect the oscillator energy shift is generated by the variable external magnetic field which determines the ω. In our system this role is played by the PQW width D. Thus, before we will proceed further we should relate the value of eigenfrequency ω to the material constants of the heterocomponents A and B that form the PQW. The quantum wells with parabolic potential profile are fabricated via continuous $A_x B_{1-x}$ variation of heterocomponents along the "z" direction. Suppose that the conduction band

edges of the heterocomponents are given by $A \to V_A$ and $B \to V_B$, then the height of the PQW potential barrier can be expressed as $\Delta V = V_A - V_B$. Having that we can determine the constant K in Eq. (1) from the condition $KD^2/8 = \Delta V$, where D is the PQW thickness. Consequently we can calculate the eigenfrequency $\omega^2 = 8\Delta V/\left(m^* D^2\right)$. In principle expression (2) resembles the eigenspectrum of free electrons being subject to the external magnetic field $B = \left(2m^2c^2\Delta V\right)^{1/2}/\left(e^2 m^* D^2\right)^{1/2}$. The only difference that in the formulation of the de Haas van Alfven effect the $k \in \mathbf{R}^1$, while in our case $k = \left(k_x, k_y\right) \in \mathbf{R}^2$. From the formal point of view one can expect this kind of spectrum when de Haas van Alfven effect in the 4D electron system is considered. In description of magnetic properties we will follow this way and will exploit the dHvA approach to the 4D system. Assuming the canonical ensemble we can write the thermodynamical potential Ω as [8,9]:

$$\Omega = -kT\int N(\varepsilon)\ln\left[1+\exp\left(\frac{\mu-\varepsilon}{kT}\right)\right]d\varepsilon \qquad (4)$$

where [8]

$$N(\varepsilon, D) = \sum_{n=0} \rho\omega\delta\left(\varepsilon - \varepsilon_n\right) \qquad (5)$$

At T = 0 the potential Ω can be estimated with help of the Maclaurin-Euler formula [19]. The contribution to the electrons that have momenta within the k and $k+dk$ $\left(k = |\mathbf{k}|\right)$ reads

$$\delta\Omega = kdk\frac{\beta m^2 c}{\pi m^* D^2}\left[\left(X - n - \frac{1}{2}\right)^2 - \left(X - n - \frac{1}{2}\right) + \frac{1}{6}\right], \qquad (6)$$

where $\left(n+1/2\right) \le X \le \left(n+3/2\right)$ and $X = A/\left(2\pi m\right)\left(m^*/\Delta V\right)^{1/2} D$, if X is outside this region then in formula (6) n should be changed by one i.e., $n \to n+1$ [19]. This means that the thermodynamical potential shows regular oscillations as the function of PQW layer thickness D. These oscillations resembles the well-known dHvA effect. Such behaviour can be easily understood, the PQW thickness determines the ω_D. In different PQW systems there are different positions of the highest $\hbar\omega_D$ level with respect to the Fermi energy. When one of the oscillatory levels crosses the Fermi energy oscillations of the electron density occur and thermodynamical potential Ω occur.

In case of arbitrary temperature T the potential Ω (4) can be estimated with the help of the Poisson formula and takes the form [8,9]

$$\Omega = \Omega_0 + \hat{\Omega}, \tag{7}$$

where the Ω_0 is the classical part of the potential Ω which does not oscillate as the function of D [8]. Contrary to Ω_0 the $\hat{\Omega}$ term behaves as [8,9]

$$\hat{\Omega} = \hat{\Omega}_0(D) + 2\sum_k \sum_{r=1}^{\infty} A_k^r \cos\left(rf_\alpha D + \phi_k^r\right). \tag{8}$$

The second term produces spatial oscillations of the free energy and is responsible for the counterpart of the dHvA effect in our system. The free energy of the system defined as

$$\hat{F} = \hat{\Omega} - \frac{1}{2\rho}\left(\frac{\partial\hat{\Omega}}{\partial\mu}\right) \tag{9}$$

also shows periodical oscillation as an function of D. Consequently if we account for the external magnetic field H the effective magnetization within the layer can be estimated as $M = -\left(\partial F/\partial H\right)_T$ shows oscillatory behaviour as the function of the PQW thickness. The density oscillations can be attributed to changes of the electron density at the Fermi level when one of the $\hbar\omega\left(n+1/2\right)$ levels crosses the Fermi energy. The oscillatory behaviour can be attributed to the change of electron population at Fermi energy

SUMMARY

In summary, we have shown that the eigenfunctions and eigenspectra of the electrons confined within the PQW and of free electrons under action of an external magnetic field (the dHvA effect) are identical. Using the mathematical apparatus elaborated for description of the dHvA effects we have found the thermodynamical potential of the mobile charge carrier (electron or hole) of the PQW system. Further, we have proved that the free energy of the system (and as result of that other characteristics) show oscillatory behaviour as the PQW thickness D is changed. Although the oscillation of the PQW in-layer spin polarization resemble oscillatory behaviour of the conventional magnetic superlattices formed of quantum wells that have rectangular profiles, its origin is completely different. In rectangular superlattices the oscillations arise form constructive interferences within the QW while in our system the oscillatory behaviour is generated by variation of electron density at Fermi level.

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