

CRITICAL TEMPERATURE IN LAYERED SUPERCONDUCTORS

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ABSTRACT

We consider Bose-Einstein condensation of preexisting Cooper pairs which are confined within quasi-2D laminar structure. We show that the spectral dimensionality α of such systems should be described within fractional-dimensional scheme. From detailed calculations results that for $\alpha > 3$ results that one would expect enhancement of critical temperature up to the limit set by the Cooper pair binding energy. We show that in the laterally modulated systems there can arise $\alpha > 3$ case while for the optimally doped YBCO system there is experimental evidence that $\alpha = 4$.

INTRODUCTION

Accumulated experimental data provide support for a widespread conjecture that superconductivity in general is a Bose-Einstein condensation of the charged Cooper pairs observed also in conventional superconductors [1]. Usually it is assumed that singlet or triplet Cooper pairs responsible for the formation of superconducting /SC/ phase exhibit 3D mobility. However, there are many systems (including copper oxides) which show laminar structure and thus restricted geometry of the system must be accounted for.. In the following we will present theoretical descriptions of Bose condensation in a systems with laminar structure. In this contribution aim we will reexamine the problem how the reduced dimensionality affects pair condensation in any laminar superconductors including thin films and SC superlattices fabricated of different materials. We must note here that to any physical system various definitions of dimension can be proposed, thus it is important to answer the question: which notion of dimension is relevant in description of Bose-Einstein condensation /BEC/. For our considerations it is important the distinction between geometrical (position) and spectral (dynamical) dimensions. While the geometrical dimension describes the mass/ion distribution the spectral dimension gives us the excitation statistics of the free electron (or quasiparticle) gas. Since the BEC depends on the low-energy excitation of the free charge carriers it is evident that only knowledge of spectral dimension is relevant in description of this phenomenon. Generally in the case of non translation-invariant structures it has been proven that spectral dimension is the best generalization of the Euclidean dimension of the system when dealing with dynamical or thermodynamical properties. The notion of spectral dimension opens the

way for research of structures which cannot be classified as a systems having integer dimension. Since there is experimental evidence that the copper oxides exhibit dimensional crossovers (e.g. in the YBCO system the magnetoresistivity measurements with indicate gradual dimensional crossover from an anisotropic 3D to quasi 2D system when the oxygen stiochiometry is changed [2]) in the following we will treat the value of spectral dimension as a continuous parameter. Guided by our intuition associated with geometrical dimension it is usually believed that the value of spectral dimension of a layered electron gas system interpolates between 2D and 3D cases. However as we will show below, there are systems for which the value of the spectral dimension exceeds three.

FRACTIONAL SPECTRAL DIMENSION

Characteristic feature of the layered SC is that the electron gas responsible for the pair formation, due to the boundary conditions at interfaces or surfaces shows both anisotropy of mobility and anisotropy of concentration. Its is evident that quasi-2D mobility of the charge carriers is essential in formation of SC state. However, approximation of the Fermi gas in a quantum well (i.e. in a layered system) by a purely 2D or 3D system is seldom a reasonable choice. The purpose of the present contribution is to formulate a simplified model of the Bose condensation in the intermediate region, when the dynamical dimensionality of the mobile charge carriers interpolates between 2D and 3D cases. We adopt the approach by He [3], who has shown that the anisotropic interactions in 3D space become isotropic ones in lower FD space, where the dimension is the Hausdorff dimension and is determined by the degree of anisotropy. Evidently, when the potential which causes the in-plane confinement is infinite, the system is purely 2D. However, in the case of finite quantum-wells the envelope functions of free electrons (holes) spread into the barrier region and partially restore the 3D character of the motion. Consequently, the system exhibits behavior, which is somewhere in between 2D and 3D.

The method by He [3] postulates that the electron quantum states are homogenously distributed in the αD k -space and a surface of constant energy is an αD spherical shell. Suppose further that the energy dispersion is parabolic $(E - E_0) \sim k^2$ we obtain the expression for the density of states in αD k -space as [3]

$$n(E)dE \approx (E - E_0)^{\alpha/2-1} dE \quad (1)$$

where E_0 is the band-gap. This means although the ionic (mass) distribution position space of dimensionality β shows no peculiarities, the density of free particle eigenstates shows (sometimes fractional) power law scaling (with effective spectral dimension $\alpha \neq \beta$) [3]. The effective spectral dimensionality of laminar system can be easily determined provided that energy spectrum of mobile particles within the layer is known. In principle it

is enough if the density of states fulfills relation (1) in a small energy window close to the Fermi energy. Extensive analytical discussion of how the effective spectral dimensionality is associated with the number of the free electron modes can be found in [4]. It is important that in many low-dimensional systems like e.g. superlattices or overlayers the vibrational as well the electron density of states, extracted from experimental data correlates with those predicted for the systems of fractional dimension $/FD/$ [3].

SUPERCONDUCTIVITY IN LAMINAR SYSTEMS

As we have assumed above the dynamical states of mobile charge carriers in some laminar systems can be described properly with help of a k -space having fractional dimension. This concern also the YBaCuO compounds for which the FD (e.g. $\alpha D = 2,03$ [5]) has been postulated. Most of the theoretical approaches to the superconductivity rely of the k -space pairing, this justifies search for SC in a system of (spectral) FD. For the use of further considerations it is not necessary to specify any peculiar mechanism of pairing. Experiments confirm that spectral FD case arise in various laminar systems involving polarons [6] (and thus bipolaronic SC, excitons [7], phonons [8], or magnons [9]. This variety of quasi-particles and interactions covers almost all mechanisms postulated for description of HTC SC (provided that real space pairing theories are excluded). Concluding we assume that the Hamiltonian that responsible for the Cooper pair formation is given by

$$H = \sum_{ks} (\varepsilon_k - \mu) c_{k,s}^+ c_{ks} + \sum_{k,k_1} V_{k,k_1} c_{k,\uparrow}^+ c_{-k\downarrow} c_{k_1\uparrow}^+ c_{k_1\downarrow} \quad (2)$$

where c_{ks}^+ the fermion creation operator labeled by k and spin s . The only difference when compared to conventional approaches is that the k -states fill the space of non-integral dimensionality. As we have mentioned above, we assume the SC transition as the Bose-Einstein condensation of preexisting boson pairs. It is well known fact that Bose-Einstein condensation produces a non-zero absolute temperature T_c , below which a macroscopic condensation emerges, only if $D > 2$. The conventional theory of boson condensation derived for systems of integral dimensionality [10] can be easily extended onto systems, which exhibit fractional spectral dimension α . The total number of bosons $N_B(T)$ in the system consists of the $N_{B,0}(T)$ ones that occupy the ground state ε_0 , ($\varepsilon_0 = 0$ in the thermodynamic limit), while the others are distributed over higher energy levels. In view of this we have:

$$N_B = N_{B,0}(T) + \sum_{k \neq 0} \frac{1}{e^{\beta(\varepsilon_k - \mu_B)} - 1} \quad (3)$$

where $\beta = 1/k_B T$ and $\mu_B < 0$ is the chemical potential. Similarly as in Eq. (3) we assume that summation goes over the k -states filling the fractional αD space. The sum over k in (3) can be converted to an integral over positive k

, where k fills the αD space with use of special formula of integration over αD space (with $2 < \alpha < 3$) [6], [8]. Applying it to Eq. (3) we obtain

$$\frac{N_{B,0}(T)}{N_{B,0}(0)} = 1 - \frac{V_\alpha}{N_B} \left(\frac{m^*}{2\pi\beta\hbar^2} \right)^{\alpha/2} \zeta(\alpha/2) \quad (4)$$

where we have accounted for the fact that at $T=0$ all boson pairs form the condensate i.e. $N_{B,0}(T=0)=N_{B,0}(0)=N_B$, ζ – is the Riemann Zeta ζ function. In calculations the parabolic energy spectrum of the quasi-particles has been assumed. It is important to note that occupation of the ground state depends only on the value of spectral dimension of the free electron system. In conventional theories the ratio V_α/N_B is treated as the inverse boson pair concentration n_B^{-1} . Such interpretation is justified provided that spectral dimension α and dimension of real space β (position space) are equal. However, in systems of FD such interpretation is not valid. Suppose, that in the system under consideration we have some characteristic length L , then the volume $V_\alpha \sim L^\alpha \sim (k_F)^{-\alpha}$. Simultaneously the volume of the system, i.e. volume filled with quasi-particles (boson pairs) can be expressed as $V_\beta \sim L^\beta$. In view of this, concentration n_B being the real space quantity reads as $n_B = N_B/V_\beta$. Distinction between this different notions of dimensionality is often missed, but as it will be shown below crucial in proper description of dimensional effects in SC. The condensate fraction falls off when the temperature is increased and eventually at T_c , the condensate vanishes i.e. $N_{B,0}(T)/N_{B,0}(0)=0$. From this condition we can derive the formula for the critical temperature T_c , as a function of the effective spectral dimension α . Inserting relation $V_\alpha \sim L^\alpha$ into Eq. (4) we have:

$$T_{c,\alpha} = \frac{m^* L^2}{2\pi k_B \hbar^2} \left(\frac{\zeta(\alpha/2)}{N_B} \right)^{2/\alpha} \quad (5)$$

Let us consider a FD system in two states, which exhibit FD α and α' respectively. Moreover let us assume that number of preexisting boson pairs is constant during this dimensional crossover. In view of Eq. (5) the hypothetical critical temperatures in both states fulfill the relation.

$$\frac{T_{c,\alpha'}}{T_{c,\alpha}} = \frac{m_{\alpha'}^*}{m_\alpha^*} \frac{\zeta(\alpha'/2)^{2/\alpha'}}{\zeta(\alpha/2)^{2/\alpha}} N_B^{(2/\alpha - 2/\alpha')} \quad (6)$$

DISCUSSION

Let us study the variation of the critical temperature $T_{c,\alpha}$ associated with the continuous dimensional crossover. We assume that in (6) $\alpha = 3$ i.e. take the 3D case as the reference system, First of all let us note that ratio $m_{\alpha'}^*/m_\alpha^*$ and $\zeta(\alpha'/2)/\zeta(\alpha/2)$ are factors of order of unity. The factor that shows strongest influence on the ratio (6) of critical temperatures in different states of the system under consideration (i.e. in states which exhibit different values of effective spectral dimension) is the last term namely the $(N_B)^{(2/\alpha - 2/\alpha')}$.

In the case $\alpha = 3$, $\alpha' = 2,8$ and $N_B = 10^{20}$ this factor can be estimated as $(N_B)^{(2/\alpha-2/\alpha')} = 10^{-1}$, while for $\alpha = 3$, $\alpha' = 2,5$ it takes value $(N_B)^{(2/\alpha-2/\alpha')} = 10^{-3}$. This means that when the effective dimension is decreased the critical temperature decreases in a very rapid manner. Contrary to the previous remark if $\alpha = 3$ and $\alpha' > 3$ one would expect an elevated critical temperature. This point is important per se independently of quantitative predictions since it allows us to draw general conclusions concerning the role of dimension in formation of SC phase. In connection with previous remarks there arises a question why the copper oxides, which are commonly believed to be quasi 2D superconductors with parabolic dispersion, exhibit so high critical temperatures?. The possible explanation is that the effective spectral dimension of the copper oxide system is higher than three. At first sight conclusion that the dimension of k-space for the boson gas confined within layers can be higher than three appears to be counter-intuitive. However, it has been proven that quasicrystals [11] and mobile quasiparticles within quantum wells [12], [13] can exhibit spectral dimension $\alpha > 3$. In classical superconductors there is no low-energy excitations, while in some unconventional SC with a line of nodes (-i.e. with zero gap along some directions, to this class belong e.g. spin singlet pairs with $d_{x^2-y^2}$ and d_{xy} symmetries [14]) are expected to have zero-field density of states $N(E)dE \approx |E - E_F|$ which can be fitted to the general formula (1) by setting $\alpha = 4$. It is important to note that specific heat measurements confirms this value of spectral dimensionality in optimally doped YBCO systems. Although the elevated spectral dimension should result in elevated critical temperature T_c there is limit up to which the T_c can be increased. In our considerations we have assumed preexistence of Cooper pairs. It is evident that pairs can exist up to the temperatures comparable with the pair binding energy. This we can conclude that when the spectral dimensions is increased the critical temperature reaches its optimum set by the pair binding energy.

REFERENCES

- [1]. M. Casas et al.: Solid State Commun. 123, 101 (2002).
- [2]. E. Osquiquill et al. : J. Phys.(Cond. Matt.) 5, A385 (1993).
- [3]. X. F. He: Phys. Rev. 43B, 2063 (1991).
- [4]. Z. Bak, J. Jaroszewicz, W.Gruhn: J. Magn. Magn. Mater. 213, 340 (2000).
- [5]. X.-G. Wen, R. Kann: Phys. Rev. B37, 595 (1988).
- [6]. A. Matos-Abiague; Phys. Rev. B65, 165321 (2002).
- [7]. P. Lefebvre, P. Christol, H. Mathieu: Phys. Rev. B48, 17308 (1993).
- [8]. F. H. Stillinger: J. Math. Phys. 18, 1224 (1977).
- [9]. V. Ilkovic, S. Tuleja: Czech. J. Phys.: 52, A57 (2002).
- [10]. M. Casas et al. : Phys. Lett. A245, 55 (1998).
- [11]. T. Jansen, O. Radulescu, A. N. Rubtsov: Eur. J. Phys.: B29, 85 (2002).
- [12]. Z. Bak: Solid State Commun. 118, 43 (2001).
- [13]. E. Reyes-Gomez et al.: Phys. Rev. B61, 13104 (2000).
- [14]. C. C. Tsuei, J. R. Kirtley: Rev. Mod. Phys. 72, 969 (2000).