Jan Długosz University of Częstochowa
Scientific Issues, Mathematics XII, Częstochowa 2007

GAINING MATHEMATICAL SKILLS BY USING 'CONCRETE' MANIPULATIVES

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Abstract. In this issue we propose the reasons why it is important to use a variety of 'concrete' manipulatives in the classroom starting from the earliest ages. And for the teachers who can see the importance of using manipulatives in their classroom, we offer some new innovative ideas.

Prologue

The elementary school mathematics curriculum devotes to content and methods on teaching mathematics, which contributes to development of creative skills, mental operations, spatial images, and also social abilities (patience, diligence, allowance, etc.) [8]. It is up to the teachers what methods they choose for gaining this goal. We think the one of the possibilities is using manipulative facilitation (further *manipulatives*).

However, manipulatives do not guarantee meaningful learning, physical actions with certain manipulatives may suggest different mental actions than those we wish students to learn. To understand the role of 'concrete' manipulatives and any concrete-to-abstract pedagogical sequence, we must define what we mean by 'concrete' manipulative.

The meaning of 'concrete' manipulative

Dr. Frobisher [4], the author of many mathematics textbooks and mathematics methodology books in Great Britain, refers to the word "*manipulative*" as some kind of classroom equipment used by teachers and children to assist with teaching and learning about the mathematical concept that it represents.

Most practitioners and researchers agree that manipulatives are effective because they are concrete. By 'concrete' they usually mean objects that students can grasp with their hands. Douglas Clements [1], in his article, refers to the wider meaning of 'concrete'. He distinguishes two types of concrete knowledge: *Sensory-Concrete* and *Integrated-Concrete* knowledge.

By *sensory-concrete* he means the knowledge when we need to use sensory material to make sense of idea. For example, at early stages, children cannot count, add, or subtract meaningfully unless they have actual things.

Integrated-concrete knowledge is build as we learn. The root of the word is 'to grow together,' it is the combination of separate ideas in an interconnected structure of knowledge. For students with this type of interconnected knowledge, physical objects, actions performed on them, and abstractions are all interrelated in a strong mental structure. For example, Jacob read a problem on a restaurant place mat asking for the answer to $3/4 + 3/4$. He solved the problem by thinking about the fraction in terms of money: 75¢+75¢ is $1.50, so $3/4 + 3/4$ is $1\,1/2$ [2]. Ideas such as '75,' '3/4,' and 'rectangle' become as real, tangible, and strong as a concrete sidewalk. Jacob's knowledge of money was in the process of becoming such as tool for him.

Therefore, an idea is not simply concrete or not concrete. Depending on what kind of *relationship* you have with the knowledge [14]. *Good manipulatives are those that aid students in building, strengthening, and connecting various representations of mathematical ideas* [1].

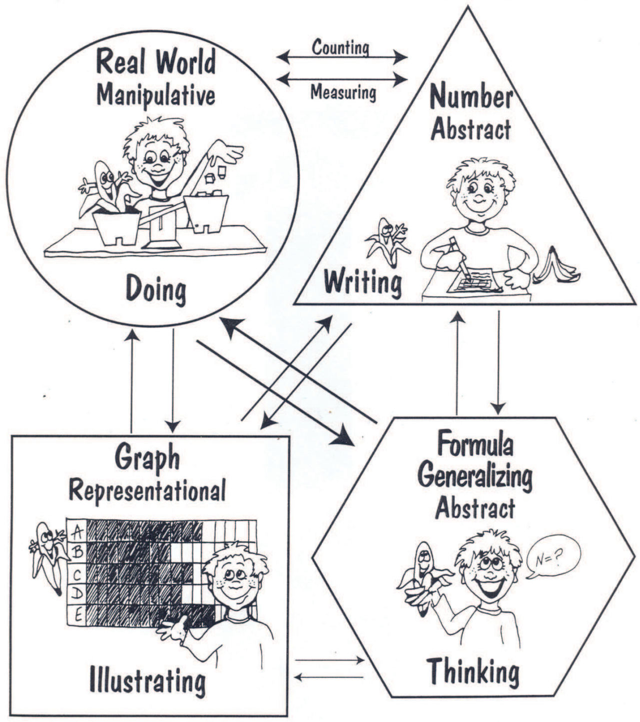
The use of 'concrete' manipulatives in mathematics education

'Real-world' mathematics has the greatest value to students. Students develop mathematical concepts and relationships best through activities in which they manipulate.

The following *Model of Mathematics* [10] serves as meaningful references for understanding the four environments in which mathematics is experienced:

The four distinct environments depicted in the models are:

- **Concrete:** experiences in the real world (circle),
- **Symbolized:** writing expressions using the symbols of mathematics and language (triangle),
- **Representational:** illustrating the real world by using drawings, graphs, and other types of picturing (square),
- **Intellectualized:** thoughtful analysis utilizing hypothesizing, drawing conclusions, making generalizations, etc. (hexagon).



The ability to translate from one environment to another is crucial. Greatest emphasis is placed on real-world mathematics, where students can see, touch, smell, feel, hear and manipulate with physical models (circle). Then are their representational counterparts, when students illustrate and draw charts, graphs, diagrams and maps (square). When students quantify and operate with numbers, they exemplify working in the triangle environment. As they problem solve, search for all solutions, analyze, formulate and discover principles, etc., they operate in the hexagon environment.

When to start with using 'concrete' manipulatives?

From an early age children gain experience with mathematics. Their exploring initially begins with tactile perception using fingers, sometimes toes and often with the mouth [7]. As parents automatically buy the toy blocks (lego, tangrams, etc.) to their children, they unconsciously help to develop their mathematical skills. By playing with constructive toys, other skills can be developed, too, e.g. manipulative skills, spatial sense, number sense, communication, etc. When children manipulate and build the objects, they question themselves: "Which brick will fit into this space? How many piece would I need? Do I need two-stud or three-stud piece of 'lego' to complete the wall?"

In our experience and after looking in several textbooks we can see that this pre-school experience is not fully continued in early years of schooling. According to this we agree with prof. Hanzel that developing mathematical thinking in pre-school and early years of schooling cannot lack the manipulation with 'real-world' objects, e.g. moving, classification and comparison. These activities are the basis for the developing number concepts meanwhile it represents initial stage for developing integer number concepts [6].

From above we can see the importance of implementing manipulatives into the mathematics education process starting at the earliest ages.

What manipulatives are efficient for mathematics education?

Piaget's point of view was that "giving no education is better than giving it at the wrong time" [9]. According to this we dare to say that not using manipulatives is better than using them in a wrong way. This fact supports Dr. Frobisher when he manifests that children become so absorbed by the manipulative that the concept it represents is so diffused that no mental image is formed [4]. Therefore, it is very essential to think of the effective and appropriate use of manipulatives that we would like to apply at mathematics education on different stage of children's development. We need to consider the age group as well as the goal we would like to claim, what mathematical skills we want to develop by chosen 'concrete' manipulative.

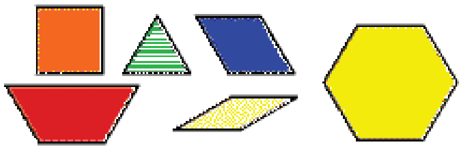
Regarding to character and consistence we can classify manipulatives into these groups:

- **toys** – building blocks, mosaics, lego, tangram, domino, etc.;
- **'real-world' objects** – fabrics, cereal boxes, clay, rods, rocks, beans, toothpicks, straws, etc.;
- **classroom equipment** – geoboard, geofigures, Attribute blocks, Base-ten blocks, Cuisenaire rods, Pattern blocks, Color tiles, etc.;
- **virtual manipulatives** – computer version of 'concrete' manipulatives, for representatives look at 'national library of virtual manipulatives' [12].

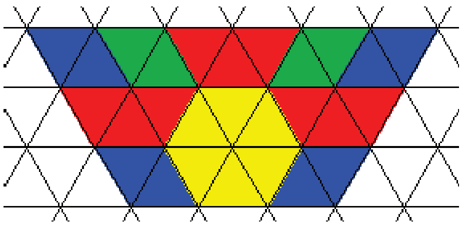
Examples of activities with 'concrete' innovative manipulative

We would like to introduce *Pattern Blocks* - the manipulative that we have first encountered when observing classes at Old Colony Montessori school (Hingham, MA) in 2006. We would like to show its importance and value at mathematics education in light of developing children's skills and their competences.

Pattern Blocks is a set of 250 blocks, it includes following geometric shapes with equal depth:

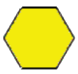


1) Activities that help to develop concept of space and symmetry:
Task: There is a mosaic on the picture. Can you locate the line of symmetry?



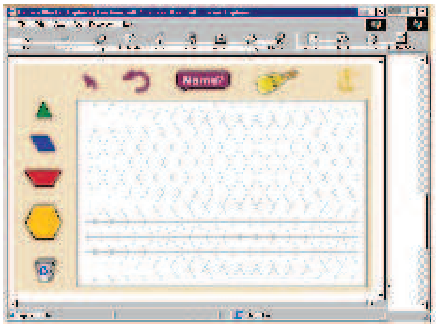
2) Activities that help to develop concept of fraction:

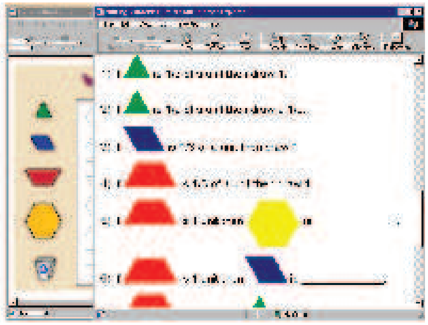
Task: If  is 1/2 of a unit then draw 1 unit.

If  is 3 units then  is _____.

3) Activities that develop mathematical vocabulary:
Architect and constructor
The architect builds the construction out of 10 blocks behind the screen. The constructor follows the architect's instruction on the other side of the screen. When they finish, the screen is taken away and the constructions are compared by both students. Then the roles are switched.

4) Activities on virtual bases [13]:
This program consists of a panel, as shown on the right, where you can drag in four different types of shapes of nicely fitting sizes - *pattern blocks* or *manipulatives*. You can place them on a triangular grid paper. Once inside the panel, you can rotate them and move them.





Conclusion

We introduced only a little fraction of the use of 'concrete' manipulatives in the classroom activities. There are lots of other manipulatives that help teachers to gain mathematics curriculum. Teachers can buy or make them through the internet or the catalogues. For other examples, as well as instruction how to create them, see the web pages

http://mason.gmu.edu/~mmankus/Handson/manipulatives.htm
http://nlvm.usu.edu/en/nav/vlibrary.html

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