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THE ROLE OF THE MILTISTAGE TASKS IN DEVELOPING THE CREATIVE ACTIVITY OF MATHEMATICS TEACHERS

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Abstract. This papier presents the results of the research carried among the mathematics teachers. These research deals with the skills of undertaking creative mathematical activity by the teachers. It also deals with the awareness of the need of developing different kinds of this activity among students. The main tool which is used and studied in the research is the multistage task.

1. Introduction to the research problem

My research deals with **the formation and developing of skills of undertaking creative mathematical activity** by mathematics teachers and the tools of provoking this activity. Developing the skills of undertaking different kinds of creative mathematical activity among teachers is the necessary condition:

- to awake their **awareness** of the necessity of formation of this activity among their students,

- to develop their **skills** of organizing the situations which favour undertaking different kinds of this activity.

Only then the teachers would form and develop effectively that activity in their work with the students.

The main tool of that formation which I use and study in my research is **the multistage task**.

However, the research connected with the observation of school reality stress a worrying aspect that developing the creative mathematical activity is neglecting. In school practice the attention is paid mainly to the students knowledge of basic concepts and skills of applying procedures. *In general,*

mathematics teaching pays little attention to the more advanced aspects of mathematical activity such as the formulation and resolution of problems, the formulation and testing of conjectures, the pursuit of investigations and mathematical proofs, and the argumentation and critique of results. While these are fundamental and current themes of mathematics education expressed in many curriculum documents across the world they still find very little emphasis in classroom practice ([3]).

2. The conception of formation of creative mathematical activity

The conception of formation of creative mathematical activity was worked out by M. Klakla ([1]). It is based on two elements:

First of them constitutes distinction of particular kinds of creative mathematical activity, which are present in essential way in activity of mathematicians. They are: (a) putting and verification of hypotheses; (b) transfer of the method (of reasoning or solutions of the problem onto similar, analogous, general, received through elevation of dimension, special or border case issue); (c) creative receiving, processing and using the mathematical information; (d) discipline and criticism of thinking; (e) problems generation in the process of the method transfer; (f) problems prolonging; (g) placing the problems in open situations.

The second element of that conception are the multistage tasks:

- which are the specific structure of series of tasks, problems and didactic situations,
- based on the problematic situations,
- connecting different kinds of creative mathematical activity with each other in the complex and rich mathematical-didactic situations,
- provide specific laboratory of creative mathematical activity for the students.

3. Information about the carried out research

To date, I carried out the pilot research and main research. **The pilot research** has dealt with the skills of undertaking creative mathematical activity by the teachers of mathematics. It also has dealt with the awareness of the need of developing different kinds of this activity among students.

The results of that research presented in [2] show that:

- Among the mathematics teachers the awareness of that what is the creative mathematical activity, and the awareness of the necessity of formation of this activity, is insufficient.

- Among the mathematics teachers it is generally erroneous believed that the creative mathematical activity develops by itself during the mathematics lesson and it does not require any special didactic endeavours, methods or tools to develop it.

- The mathematics teachers do not have experience and skills of undertaking that activity.

The aims of **the main research**:

1) Recognition:

- the awareness of the necessity of formation of creative mathematical activity;

- the initial skills of undertaking creative mathematical activity.

2) Verification of the put hypothesis, i.e. that the multistage tasks can be used as a tool:

- to introduce the teachers of mathematic into given creative mathematical activity (introductive means);

- to develop the skills of undertaking the creative mathematical activity among teachers of mathematics (developing means);

- to form among the teachers mathematics the awareness of the need to develop creative mathematical activity and the skills of provoking this activity among students (awareness means);

- to diagnose the skills to undertake the given kind of creative mathematical activity by teachers of mathematics (diagnostic means).

4. The description of the process of the main research

The group of 7 teachers of mathematics (of gymnasium and high schools) has taken part in the series of workshops from March to September 2006. The workshops, concerning three multistage tasks, were organized as a part of the Professional Development of the Teachers-Researchers (PDTR) project, during the mathematics course.

Before the series of workshops, the diagnosis of the skills of undertaking the creative mathematical activity and the awareness of the need to develop this activity among students had been carried out (the research tool developed during pilot research and the set of two open tasks). After the series of workshops, the next diagnostic research was carried out (the research tool - the questionnaire and the same set of two open tasks).

5. The fragment of analysis of solving the set of open tasks

The teachers received the set of two open tasks both before (solution I) and after the workshops (solution II). The aim was to diagnose their skills of undertaking the creative mathematical activity. There is one of that tasks:

The Hippocrates' lunes (Fig. 1) *constructed on the right triangle ABC with the right angle C are planes contained by the arc ACB of a circumscribed circle and the arcs of semicircle on diameters equal the lengths of two adjacent sides BC and AC and the centers in the centers of these sides.*

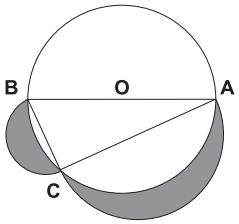

Fig. 1: The Hippocrates' lunes

Formulate some relevant questions connected with the situation presented in the picture and try to answer these questions.

I will present the solution I and II by one of the chosen teachers, who has taken part in the workshops.

The solution I (before the workshops):

The situation presented in the task was familiar for the teachers. The chosen teacher, like majority of the researched group, put only the question about relation between area of the Hippocrates' lunes and area of the right triangle ABC, referring to the known for her property.

The solution II (after the workshops):

She started the solution with the questions: *What is the area of the Hippocrates' lune? What is the sum of area of the Hippocrates' lunes?* And then she proved the relation: *The sum of area of the Hippocrates' lunes equals the area of triangle ABC* (alike in solution I). Afterwards she was prolonging the task modifying the initial situation in the ways presented in Fig. 2, Fig. 3, Fig. 4.

$P_I + P_{II} = ... = P_{\square} - P_{\triangle ABC}$,

where $P_{\square}$ means the area of the square with the side AB.

$P_I + P_{II} = ... = P_{\square} - \frac{1}{2}P_k + P_{\triangle ABC}$,

where $P_{\square}$ means the area of the square with the side AB and P_k means the area of the circumscribed circle.

$P_I + P_{II} = ... = 2P_{\triangle ABC} - \frac{1}{2}P_k$,

where P_k means the area of the circumscribed circle.

The first direction of prolonging is the replacement the circles with the squares (Fig. 2). The second direction of prolonging is building polygons (squares, right triangles) on two adjacent sides BC and AC of the triangle

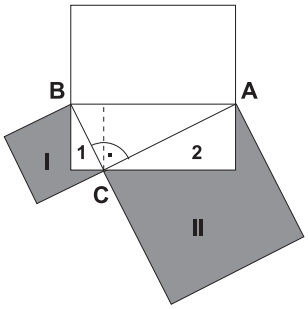

Fig. 2: The teacher's solution II, part 1

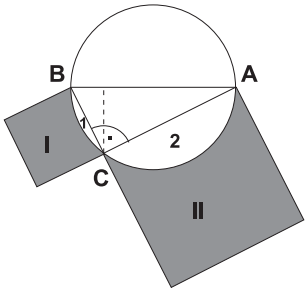

Fig. 3: The teacher's solution II, part 2

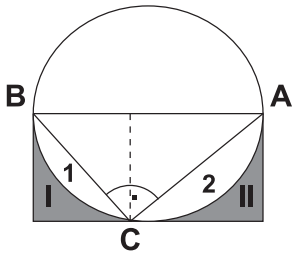

Fig. 4: The teacher's solution II, part 3

ABC inscribed in a circle (Fig. 3, Fig. 4). The work of the teacher consists in modification of the initial situation and verifying if it is analogous, to the initial situation, relation between areas of the figures I and II and the area of the triangle ABC, where P_I, P_{II} are the areas of the figures built on two adjacent sides BC and AC of the triangle ABC after substraction the areas 1 and 2 (rights triangles - Fig. 2 or parts of the circles - Fig. 3, Fig. 4).

During the solution II the teacher undertook several kinds of creative mathematical activity: she put hypotheses and made an attempts to verify them; she used the transfer of the method of solutions onto similar, analogous issue, she was prolonging the task. The researched person started to notice the potential of that situation, value of the open task and she used the possibility of free leading her solution, not to be confined only to the known by her relation. The teacher formulated untypical problems characterizing of creativity. It can be evidence of that she approaches more flexibly to the mathematical tasks and also it has been revealed visibly the skill of undertaking by her the creative mathematical activity.

The researched person declared in the questionnaire after the series of workshops in following words:
During the workshop *I have learned that the tasks of this type can be used in the school, they are interesting and educational for the pupils as well as for the teacher. Solving this type of the tasks gives lots of fun.*
Considered problems aroused my interest - the first task I used on the additional lessons of mathematics (both in the first and third class of gymnasium).
The tasks of this type you can use with the work with gifted students and some elements of those tasks also during typical lessons of mathematics.

As long as before the workshops, the researched person declared that she does not consider with the pupils the tasks which are prolongation of a given task and that she rarely solves the open tasks with the pupils, so during the series of workshops she started to use some fragments of multistage tasks in her didactic work. It can be evidence that her awareness of the need to develop creative mathematical activity among students has been rosen. It can be also evidence that a change of her attitude in the direction of taking an action oriented to developing creative mathematical activity among her students has been ensued.

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