Jan Długosz University in Częstochowa
Scientific Issues, Mathematics XVI, Częstochowa 2011

DIDACTIC PC GAME AS A TOOL OF CURRICULUM ENRICHMENT OF MATHEMATICALLY GIFTED STUDENTS EDUCATION

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Abstract. Curriculum enrichment principle is one of the approaches in education of mathematically gifted students. Usage of didactic computer games could be one of the possibilities of accomplishment.

1. Introduction

Curriculum enrichment principle is one of the ways of creating proper conditions at school, that is to say by modifying educational content, helping the gifted pupil's to develop their capacities in an optimal way. It brings enrichment of knowledge, interests and abilities beyond the regular curriculum.

The enrichment should be divided in three levels (broadening, deepening, and enriching the curriculum) and should be focused mainly on developing higher order thinking processes, self-reliance in problem solving and creativity. The enriched curriculum should follow the abilities, knowledge and needs of a gifted pupil [1, 3].

Current curricular documents designated for elementary schools support using computers for education of mathematics. They also constitute problem solving and solving of nonstandard application problems as one of the four main topics of mathematical education at elementary schools. A nonstandard application problem is called a problem that has to be solved by using not frequent methods or different methods that pupils know in their classes. One of the ways of reaching this intent is the application of didactic computer games to mathematics teaching.

There are a lot of programs or didactic games based on a mathematical problem, so we picked one of them.

2. Didactic computer game "4COLORS"

This is one of the older (a DOS program) but very usefull game based on the four colour theorem. It states that four colors are sufficient to color any map so that regions sharing a common border receive different colors.

It is very easy to demonstrate the mathematical induction proof to elementary school pupils by using the "4COLORS". We have to keep in mind the complexity of this kind of proof. During solving the next problem we can see that the pupils gifted with mathematics can comprehend. Mathematical induction is a deductive process which is specific for mathematicians but not for pupils (the mathematician in the first steps of his heuristic strategy solves using induction; in the case he finds "something" he proves by inference).

Let us solve a problem:

Is it possible to color all of the regions in the plane devided by the given number of lines using only two colors, so that regions colored by the same color share only points – vertices? If it is possible, what is the number of lines that accomplish requirements?

There is a geometric character of this problem, problem solving by using "4COLORS" is very rational. We have to draw a new map. Because of some software problems we have to draw a rectangle at first. That is the plane. Then draw a line in this rectangle. Now the plan is divided in two areas. It is easy to color this map by using only two colors. Because of the software problems we have to use the third color to color the area outside the rectangle. After that the program displays that we managed the task. Then we can draw a new map (a new rectangle) with two lines inside. The plane is divided into four areas. Again, there is no problem to color the map using only two colors. We are able to continue with drawing three lines, four lines, etc. and each time we are able to solve the problem. At this moment the pupil could dispute. What if this four lines would be placed in a different way? How many qualitatively different situations are possible when we place four lines inside the plane? Do we have to solve each situation particularly? Considering this questions, this stadium is very suggestive for each pupil because they notify: I am not evidently able to verify my hypothesis (there is always a solution for any number of lines despite their positions in a plane) in the way of solving the problem, for example, for $1, 2, \dots, 10$ lines. That is pupil's dilemma that allowes us to perform mathematical induction. We have to notify that the most of statement characters valid for all natural numbers introduced to pupils are easily provable. In the most cases the pupil "proves" for $n = 1, 2, 3$ and then he declares: it is proved for all natural numbers.

Mathematical induction (sometimes called full induction) is connected with mathematical axiomatics, so it is complicated for beginners. Hence we can present it to the pupils as a fact:

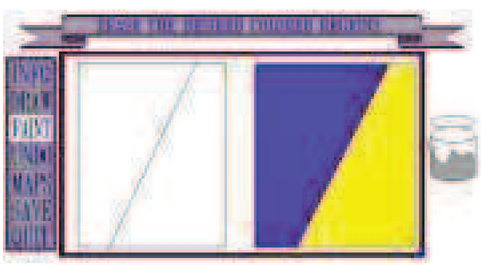
If any mathematical statement is true for some natural number k and if validity for n + 1 follows from the validity of the statement for any natural number n, then the statement is valid for all natural numbers greater than or equal to number k.

Now we reword the first task:

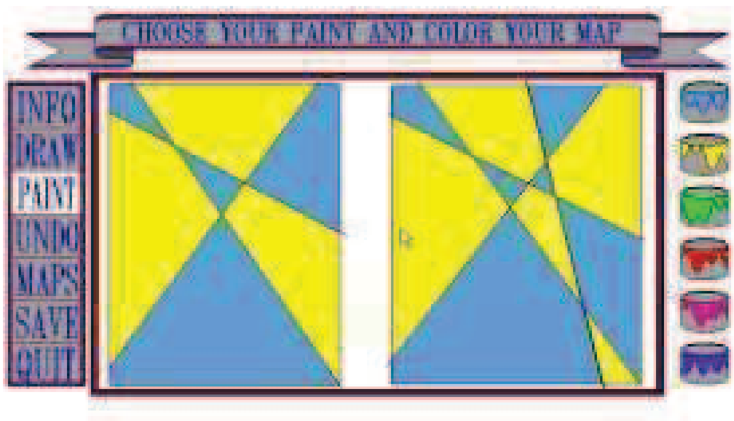
Demonstrate that it is possible to color all areas of a plane so that any pair of areas colored by the same colur contacts only in their vertices, without regards to the number of the lines dividing the plane.

The solving process is demonstrated by using "4COLORS".

1. The plane is divided by one line and these two areas are colored by different colors. The statement is valid for $k = 1$.

2. Let us draw two rectangle planes in one picture. Draw some lines into the left plane and then draw the lines the same way into the right plane and add one more line. Now we have two maps. Color by two colors the left map. In this case we manage to do it – we do not have to be affraid of failure caused by different positions of the lines. Why it is so we demonstrate later. So in the left map the statement is valid for n. Let us color the right map. The added line devides the rectangular plane into two areas. One of them is colored by the same color as in the left map. The other area has to be colored by the other color used in the left map. The result has to be the requested coloring.



In the case we draw the "added line" from the right map into the left colored map, there would be some areas where the "added line" goes through (there are the requirements broken) and some areas where does not (there are the requirements accomplished). But there is simple solution how to accomplish the requirements in the areas with "added line". We have to use the other color in one of the two areas divided by the "added line" and colored by the same color.

In the left map there are $n + 1$ lines, on the ground of verity of the left map (verity for n) we proved verity of the right map (for $n + 1$). Finally, the "added line" was placed randomly and so we have solved the problem of different positions of the lines – it does not matter at all!

On the ground of the full induction, the statement is valid for any random natural n.

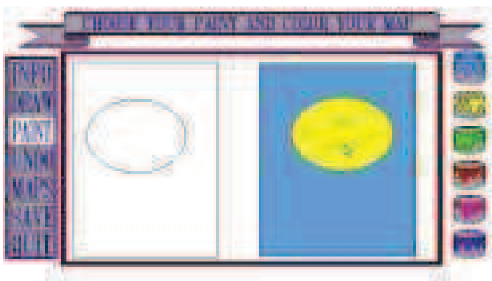
The usage of "4COLORS" for this problem solving is very apposite because pupils have an oppurtunity to go through the second induction step many times, as long as they understand its logical structure. After that they can understand deductive procedure of mathematical induction proof.

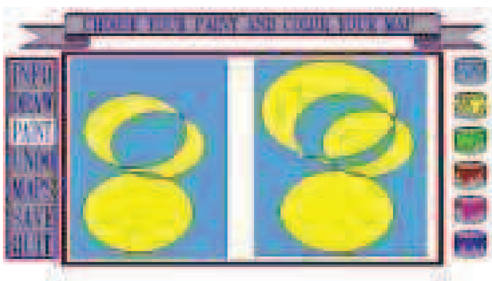
They also understand that validity for k (in this case $k = 1$) is basic; the validity of statement for $n + 1$ following from the validity of statement for n does not suffice in order to verify the validity for $n \geq k$ or for all n. At this moment we can give the pupils an idea of the mathematical induction by using domino: let us have a row of domino stones, the induction step means that if the first stone falls, the next stone is hit and falls too (the step means that the first (or kth) stone really falls).

In the task the crucial facts are that a line devides a plane into two disjoint areas and two lines can share only one point. So if we mention this facts, we can generate the next task.

Demonstrate that all the areas of a plane divided by a random number of circles/ellipses are colorable by two colors, so that two areas colored by the same number contacts just in vertices.

Solution of this problem is analogous to the solution of the first task and is clearly shown in Figures.





Acknowledgements

The present paper was written with the support of the FRVŠ 1611/2011 project called "The Education of Gifted and Talented Students".

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[4] "4COLORS" available on World Wide Web: <http://www.math.ucalgary.ca/~laf/colorful/games.html>