GEOMETRICAL IMAGINATION
AND KNOWLEDGE OF TRIANGLES
AT ELEMENTARY SCHOOLS

Ludmila Benešová, Jakub Tláskal,
Josef Molnár, Jana Slezáková

Department of Algebra and Geometry, Faculty of Science
Palacký University in Olomouc
17. listopadu 1192/12, 771 46 Olomouc, Czech Republic
e-mail: lidabene@seznam.cz   jakub.tlaskal@seznam.cz
e-mail: josef.molnar@upol.cz   slezakov@seznam.cz

Abstract. A pilot study concerning rudimentary knowledge of triangles and spatial imagination is submitted to 7th-grade pupils of an elementary school and students of the same age at a grammar school. The theory is important but practical problems are more interesting for students. Problems in spatial imagination allow a teacher to specify the level of students' mathematical abilities. The aim of this pre-research is to compare knowledge of pupils of an elementary school and a grammar school.

Nowadays more practice than theory is getting to the center of interest in education at schools. Pupils need to find some bridges between theory and practice.

In mathematics, geometrical problems are a good connection between theory and practice. Pupils use their theoretical knowledge to construct concrete tasks. At first, many questions in geometry seem to be theoretical and pupils need to apply a spatial imagination to resolve these problems.

27 pupils of a grammar school in Lanškroun and 32 pupils of an elementary school in Letohrad were tested on 20 April 2011. Pupils were of the same age (2nd grade of the grammar school, 7th grade of the elementary school). Pupils answered ten questions concerning basic geometrical knowledge about triangles. The results of the research are summarized in the tables below with the following symbols used:

+ ... correct answer

− ... incorrect answer

0 ... missing answer

Question 1: The point of intersection of medians separates medians:

a) exactly in the middle

b) shorter parts of medians are halves of longer parts

c) noway, because the medians do not intersect themselves

d) without rules

Results of question 1	7th grade of the elementary school	2nd grade of the grammar school
+	16%	33%
−	81%	67%
0	3%	0%

Question 2: The midpoint of the inscribed circle of a triangle lies in the point of intersection of:

a) bisectors of angles

b) bisectors of sides

c) altitudes

d) medians

Results of question 2	7th grade of the elementary school	2nd grade of the grammar school
+	53%	30%
−	47%	70%
0	0%	0%

Question 3: The altitude of a triangle is:

a) the straight line going through the vertex of a triangle and orthogonal to the opposite side

b) the line with outside points – the vertex of a triangle and the heel of orthogonal going through the vertex of a triangle to an opposite side

c) the length of a segment as in item b)

d) the line with outside points – the vertex of a triangle and the midpoint of the opposite side

Results of question 3	7th grade of the elementary school	2nd grade of the grammar school
a (+)	65%	44%
b (+)	13%	19%
c (+)	3%	0%
d (−)	16%	33%
0	3%	4%

The third question is a special question. Three answers are correct. Only answer d is incorrect.

Question 4: The midpoint of the circumcircle of a triangle lies at the point of intersection of:

a) bisectors of angles
b) bisectors of sides
c) altitudes
d) medians

Results of question 4	7th grade of the elementary school	2nd grade of the grammar school
+	34%	37%
−	63%	63%
0	3%	0%

Question 5: The point of intersection of altitudes separates altitudes:

a) exactly in the middle
b) noway, because the altitudes do not intersect themselves
c) shorter parts of altitudes are halves of longer parts
d) without rules

Results of question 5	7th grade of the elementary school	2nd grade of the grammar school
+	22%	26%
−	66%	74%
0	12%	0%

Question 6: There are two interior angles of 40° in a triangle. This triangle is:

  a) equilateral

  b) obtuse-angled and isosceles

  c) obtuse-angled and equilateral

  d) acute-angled and isosceles

Results of question 6	7th grade of the elementary school	2nd grade of the grammar school
+	16%	56%
−	84%	41%
0	0%	3%

Question 7: The sum of the measure of acute angles in a right-angled triangle is:

  a) 80°

  b) 90°

  c) 100°

  d) 180°

Results of question 7	7th grade of the elementary school	2nd grade of the grammar school
+	44%	67%
−	56%	33%
0	0%	0%

Question 8: It is not true that in an obtuse-angled isosceles triangle:

  a) one of the medians is perpendicular to the opposite side

  b) it is axially-symmetrical

  c) a bisector of one of angles is perpendicular to the opposite side

  d) the midpoint of the circumcircle of a triangle is the same as the midpoint of inscribed circle, the centre of gravity and the point of intersection of altitudes

Results of question 8	7th grade of the elementary school	2nd grade of the grammar school
+	25%	26%
−	56%	70%
0	19%	4%

Question 9: The central transversal line of a triangle is:

a) the line with outside points – the vertex of a triangle and the midpoint of its opposite side

b) the line which is the connecting line of the midpoints of two sides of a triangle

c) the line which is the connecting line of the vertex of a triangle and the midpoint of its opposite side

d) the line going through the midpoint of a triangle

Results of question 9	7th grade of the elementary school	2nd grade of the grammar school
+	25%	48%
−	47%	45%
0	28%	7%

Question 10: The measure of interior angles in the obtuse-angled isosceles triangle is:

a) different, depending on the size of sides

b) two of these angles are the same and the third one is the complement to 180°

c) we are not able to determine it

d) 60°

Results of question 10	7th grade of the elementary school	2nd grade of the grammar school
+	44%	89%
−	47%	11%
0	9%	0%

The pupils of the grammar school were more successful than the pupils of the elmentary school in the majority of questions. The better group was formed by the pupils of the grammar school, but there was nobody with full score – 10 points, while at the elementary school there was one pupil with 10 points.

The group of pupils of the elementary school was better only in two questions (number 2 and 3). The worst answers in both groups were to question 5. The least good answers at the elementary school were to questions number 1 and 6 and at the grammar school to questions number 5 and 8. Mistakes in

question 8 occurred probably because this question was in negation. To question number 3 there were three good answers, but nobody marked all of them. The majority of pupils chose the answer a (an altitude of a triangle is a straight line going through the vertex of triangle and orthogonal to the opposite side).

Different results at two schools are influenced by more factors. Some problems could be caused by inattention of the part of pupils, nontraditional form of validation of knowledge, and lack of maturity for the correct, independent and accurate understanding of basic concepts of plane geometry. Each of the schools uses different school texts and there are different teachers, too. The better results had pupils of a grammar school, what was expected because a grammar school is the selective school.

Acknowledgements

References

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