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# Popper and paraconsistency

**1.** Paraconsistent logic was introduced in order to provide the framework for inconsistent but non-trivial theories. We call a theory inconsistent if it contains as theorems a formula and its negation at the same time, and we call it trivial if every formula that can be expressed in the language is its theorem. By a logic we understand a set of theorems.

In classical logic there is the following law

$$A \to (\neg A \to B)$$

which says that once we accept two contradictory statements, then we have to accept any possible statement. The denial of this law is considered the trademark of paraconsistent logic. Without it we can accept two contradictory formulas $A$ and $\neg A$ and we do not get as a theorem any arbitrary formula $B$, that is, the acceptance of a contradiction does not entail triviality.

The roots of the idea of paraconsistency are claimed to be in the works of two philosophers who almost simultaneously but independently came to similar conclusions. J. Łukasiewicz in 1910 thought about logic without another principle of classical logic, the so-called law of non-contradiction, that is

$$\neg (A \ \& \ \neg A).$$

The rejection of this law, according to Łukasiewicz, allows us to accept contradictions. So-called non-Aristotelian logic obtained this way does not have to be scary, at least from the scientific point of view. Łukasiewicz suggests in his 1910 book that the presence of contradiction is no obstacle in assertion of experimental facts and makes no difficulties in deductive and inductive way of inference, since inconsistent thinking does not exclude rational thinking.

Between 1911 and 1913 N.A.Vasiliev carried out very similar investigations at the University of Kazan. He believed that rejection of the law of non-contradiction would result in something that can still be called logic, as non-

Euclidean geometry is still called geometry (n.b. Lobachevski made his discoveries also at Kazan University). His non-Aristotelian logic turned out to have interesting properties which made A.I.Arruda to formalize them in an axiomatic way in her paper of 1977.

First formal system of paraconsistent logic was introduced in Poland in 1948. Since then thousands of papers have been published in this field. The last three decades were especially productive, bringing a rich variety of results obtained all over the world. Paraconsistency bacame one of the fastest growing branches of logic with its informal centres in Poland, Australia and Brazil (see d`Ottaviano, 1990).

The main motivation for paraconsistent logic is its usefulness as a framework for inconsistent but non-trivial theories. There are numerous examples of theories of this kind that have occurred in the history of science. The theory of infinitesimals in the early theory of calculus, the naive set theory, Meinong`s theory of impossible objects, or Everett-Wheeler theory of quantum mechanics used to be the favourite examples put forward by the logicians in the area. Nowadays the main field of interest for paraconsistent logicians seems to be in computer science. The so-called expert systems, the inconsistent databases or knowledgebases require brand new frameworks for formal reasoning that tolerate inconsistencies. That is exactly what paraconsistent logic is for. In 2001 in Las Vegas, USA, took place the US congress on paraconsistency held together with the international conference on artificial intelligence which proves how important these aspects of logic became in the last few years.

**2.** K.R.Popper touched on the problem for the first time in his paper "What is dialectic?" In the first part of this paper Popper sketches out a system of logic that allows contradictions and does not allow every statement. He takes a closer look at some of the classical rules of inference analysing their behaviour when they are applied to inconsistent premisses. His conclusions are by and large negative about the possibility of a plausible system of that kind of logic. In his exact words they amount to what follows.

The question may be raised whether this [i.e. $A \rightarrow (\neg A \rightarrow B)$] situation holds good in any system of logic, or whether we can construct a system of logic in which contradictory statements do not entail every statement. I have gone into this question, and the answer is that such a system can be constructed. The system turns out, however, to be an extremely weak system. Very few of the ordinary rules of inference are left, not even the Modus Ponens. (...) In my opinion, such a system is of no use for drawing inferences although it may perhaps have some appeal for those who are specially interested in the construction of formal systems as such.

Today, having the experience of a few decades, we can correct Popper`s claim. He is definitely right saying that a system of logic in which Modus Ponens fails is of no use. But he is obviously wrong predicting that there are no strong paraconsistent systems in which Modus Ponens is valid. The calculi soon to be discovered proved the opposite. What Popper was thinking of was one and only paraconsistent formal logic that (because of its required properties) had to be extremely weak. Today we have tons of paraconsistent calculi; some of them are indeed fairly weak, but there are calculi that are very close to classical logic.

Negation considered by Popper in his paper on dialectic is a sort of dual to intuitionistic negation. For this negation operator both $A$ and $\neg A$ hold but it is not possible to conclude an arbitrary $B$ from $A$ and $\neg A$. In a more explicit way the idea of dual to intuitionistic logic was taken up in his 1948 paper. That was his another attempt to explore the logic of contradiction. The same idea was formalized later on by N.D.Goodman (1981) and I.Urbas (1996). Goodman applied his anti-intuitionistic logic to anti-intuitionistic set theory in which Russell`s paradox was derivable. Unfortunately this logic did not contain an adequate notion of conditional.

D. Miller (2000) proposed a dualized version of intuitionistic logic as the logic appropriate to falsification, contrary to earlier suggestions that intuitionistic logic is the correct logic for this purpose. In this system the law of excluded middle holds and the law of non-contradiction fails; the appropriate semantics is obtained by straightforward dualization of the standard Kripke semantics. It is undoubtedly a paraconsistent logic.

Popper`s paper on dialectic (published in 1940) was presented by him to the seminar at Canterbury University College in New Zealand back in 1937. At that time Łukasiewicz`s and Vasiliev`s works existed only their abstract versions edited in German. Since they weren`t concerned with the main stream of logical investigations of that time (and since the summary of Vasiliev`s results was practically unavailable) nobody paid a proper attention to them. It is almost cartain that Popper did not read the abstracts in the 1930s.

It is a shame that Popper`s contribution to the subject of paraconsistent logic has not been properly recognized so far. It should not matter that he only glimpsed the possibility of a formal paraconsistent logic and was more sceptical rather than enthusiastic about its content. In his paper there is an evident preconception of a new logic. Although only mentioned his very idea of such a logic was fresh and original. Since it is very unlike that Popper was familiar with the abstracts of Łukasiewicz`s and Vasiliev`s works, he should be undoubtedly regarded as an independent forerunner of paraconsistency.

**3.** Popper`s remark about an extremely weak system triggers the curiosity about the possibility of building a strong one. On the other hand most of the existing systems are fairly weak. The basic requirements of paraconsistency limit in a dramatic way the space for formal manoeuvres. Once we eliminate the undesired laws we have to eliminate the whole classes of formulas, among them those that are crucial for elementary reasoning. There are paraconsistent calculi that have among those formulas that contain negation only the double negation law (the non-intuitionistic one) or the law of excluded middle.

Relatively rich are the systems of paraconsistent logic that are based on relevance logics. But the systems obtained this way lack some of the important features already in their positive (negation-free) parts, although there is no good reason for emasculating them in such a way. The failure of $A \to (B \to A)$ for example has no convincing motivation from the paraconsistency viewpoint.

Some spaces between well-known logics (understood as sets of tautologies) indicate the areas of special interest in the search for paraconsistent calculi. Paraconsistent extensions of positive Hilbert calculus that are proper subsets of intuitionistic logic are still not entirely explored, although they are not very promising for those who get impressed by the size. Especially interesting is the upper part of the space between positive Hilbert calculus and classical logic. Proper subsets of classical logic, and as large as possible are the candidates that seem the most interesting the (see my paper of 1996).

This way within the multitude of sets of tautologies we search for the largest ones. Those we are especially interested in are called maximal and defined as follows.

A paraconsistent calculus **M** is maximal if there is no paraconsistent calculus **N** that properly includes **M**.

One of the ways of proceeding is to preserve all the connectives in their classical forms except for negation. While defining the semantic clauses for them it suffices to care for semantic clause for negation only. It is probably the quickest way to obtain a paraconsistent calculus.

One of the first paraconsistent calculi we have come across is the one whose negation satisfies the following condition

$$V(\neg A) = 1 \quad \text{for all } A \text{ and all evaluation functions } V$$

that is, strangely enough, negation of every sentence is always true. This condition combined with the standard semantic conditions for other connectives determines the set of tautologies that forms a paraconsistent calculus. But a closer look at it brings some disappointment. It turns out that this calculus contains some uninteresting formulas, i.e. those that fail to be classical tautologies (e.g. $\neg A$). In order to get a „decent" paraconsistent calculus we have to take an intersection of this set of tautologies with the set of classical tautologies. This way obtains an extraordinarily strong calculus. It contains a double negation law, de Morgan laws, excluded middle, and some versions of contraposition. It came as no surprise to us when the same calculus was obtained in a different way by J.-Y. Beziau and N.C.A. da Costa (1993), and later on was proved to be a maximal paraconsistent calculus (M. Nowak, 1999). Moreover it is an axiomatizable calculus as well.

While writing about maximality it is hard to ignore the results obtained by D. Batens. His paper of 1980 is probably one of the most important in this subject. Batens defines the whole class of paraconsistent calculi of which some are maximal (he also considers various notions of maximality). He axiomatizes them and proves the completeness theorems using an original and elegant method. It is the first systematic account on the notion of maximality in paraconsistent logic.

Searching for maximal paraconsistent calculi is reaching for the limits of the hierarchy of paraconsistent logics. They are still unknown. Some of them might be of great importance for computer science. Especially those that are axiomatizable, since they may be useful for the artificial intelligence business.

Popper`s remark concerning the weakness of the system of paraconsistent logic issued a challenge that it was hard to ignore. It was his paper on dialectic that sparked our interest in how strong paraconsistent calculi can be. In our joint research with T. Skura we are looking for the answers to some basic questions that arise (e.g. about the number of maximal logics).

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