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Queueing systems with random length demands

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The determination of the amount of buffer space required for keeping information of messages in service or waiting for service is one of the most important problems to be solved when designing information processing and communicating systems [1,9]. We suggest to use queueing theory methods for solving the problem. But classical queueing theory is not sufficient for this, because the solution depends much on two main factors: 1) each real message (or demand in queueing theory terminology) has some random length; 2) the processing time of the message depends on its length.

If we want to determine the message length as the amount of memory space required for message information storage, the solution of the problem presupposes the definition of statistical characteristics of the summarized messages volume in the system at the given time moment.

The queueing theory is the mathematical instrument rather adequately representing information processing and communicating systems. To obtain summarized volume characteristics it is necessary to presume that the queueing system has some amount of memory space V, expect for a given number of waiting places (in some cases may be $V = \infty$). Let $\sigma(t)$ be the demands volume (the total sum of demands lenghts being in service and waiting for service) at time moment t. At arrival moment τ the information of demand with lenght x is dropped into the memory immediately if $V - \sigma(\tau - 0) \geq x$, at this moment the summarized volume increases by x. If $V - \sigma(\tau - 0) < x$, the demand will be lost. At the end of the service of a demand its information leaves the memory immediately, and is doing so the summarized volume decreases by the demand length. Let ζ be the demand length and ξ its service time.

Let ξ depends on the length ζ only. Let $F(x,t) = P\{\zeta < x, \xi < t\}$ be the common distribution function of the random variables ζ and ξ. Let $L(x) = F(x,\infty), B(t) = F(\infty,t)$ be the distribution functions of the random variables ζ and ξ respectively.

It is obviously, that in real systems inequality $0 < V < \infty$ takes place, i.e. $\sigma(t) \leq V$. But in the case of dependencies between ζ and ξ we can't as a rule determine characteristics of $\sigma(t)$ process exactly. Generally we can obtain such characteristics using the next mathematical queueing models: 1) models of queueing systems in which $V < \infty$ and random variables ζ and ξ are independent, i.e. $F(x,t) = L(x)B(t)$; 2)models of queueing systems in which $V = \infty$ and ξ depends on ζ, i.e. $F(x,t) \neq L(x)B(t)$. The case of systems in which ζ and ξ are independent and $V = \infty$ is trivial. We can use the models of the first class in the case of real systems in which dependence between ζ and ξ is inessential. For these systems we can obtain characteristics of demands number in the system and determine probability of demands losing.

If $V = \infty$ the determination of these characteristics is impossible as losses are absent in the system (when the number of waiting places and other parameters have no restrictions). In this case we can obtain characteristics of $\sigma(t)$ process. After that we can value characteristics of losing for the system with $V < \infty$ using for example the next inequality [2] for probability of losing p_l :

$$p_l \leq 1 - \int_0^V D_\infty(V - x)dL(x),$$

where $D_\infty(x)$ is the distribution function of stationary summarized volume (if the limit $\sigma(t) \Rightarrow \sigma$ exists in the sence of a weak convergence when $t \to \infty$) for the case of $V = \infty$. Thus the analysis of the both classes of models has a practical sense.

Models of the first class with some additional restrictions were analyzed in [3], models of the second class were analyzed in [2,4-7]. In [2,8] priority queueing systems were discussed.

Note, that models of the second class are more interesting from the mathematical point of view. One of the most general results for such models is an analog of the famous J. Little's formula. So, if we have usual stationary queueing system without restrictions and renewal of service, the next relation takes place [7] for the first moment $E\sigma$ of the random variable σ:

$$E\sigma = \lambda[w_1 E\zeta + E(\zeta\xi)],$$

where λ is an intensity of an entrance flow of demands, w_1 is a first moment of the queueing time, $E(\zeta\xi)$ is a joint $1+1$–moment of random variables ζ and ξ.

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