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THE PROBLEM OF DISTRIBUTION OF INDIVISIBLE GOODS

Within the scope of this work the discussion of linear model of economic problem of distribution of indinsible goods is performed for the case in which the total demand is higher than the total supply. In the work the transport cost minimization is not considered but the problem is adapted to application of the known solution (see [1]) by the change of the demand. Changing the demand δ into δ^* the satysfying of the needs of market was considered to play the main role proportionallly (in certain meaning) to Б and the minimization of costs to be not so important. The problem of distribution is considered within the established period of time, e.g. a year. In the paper the following denotations will be used.

N – set of natural numbers,
i, (j) – index integer varying from 1 to the fixed $m \in N$, $(n \in N)$, respectively,
$a=(a_j)$ – n – dimensional vector,
$b=(b_i)$ – m – dimensional vector,
$(a \cdot b)$ – scalar product of vectors a, b,
e – vector of unities of dimension of matrix columns that follows it,
$[\alpha]$ – integer part of number α,
$\delta=(\delta_j)$ – market demand $R=(R_j)$, where $\delta \in N \cup \{0\}$,
$\text{Б}=(\text{Б}_i)$ – factory supply $F=(F_i)$, where $\text{Б} \in N \cup \{0\}$.

We assume that the sings: $+, -, \leqslant, \geqslant, =, []$, for vectors are referrend to their cordinates, e.g.: if $a=(a_j)$, then $a+e=\delta$ means that $\forall_j \; a_j+1=\delta_j$.
The solution of transport problem (and hence, distribution) exist only when

$$(e \cdot \text{Б}) \geqslant (e \cdot \delta) \tag{1}$$

(see ref [1]).
We consider first the case when the condition (1) is not satisfied.
Denote:

$$\vartheta=(e \cdot \text{Б}) \cdot (e \cdot \delta)^{-1},$$
$$\bar{\delta}=[\vartheta \cdot \delta], \tag{2}$$

hence, $\vartheta<1$ and

$$(e \cdot \bar{\delta})=(e \cdot [\vartheta \cdot \delta]) \leqslant \vartheta \cdot (e \cdot \delta)=(e \cdot \text{Б}).$$

Conclusion 1. For the demand $\bar{\delta}$ the solution of the transport problem can be found by the use of the commonly methods.

The productivity of the factory F is not used completely.
Introducing vector $r=\vartheta\cdot\delta-\bar{\delta}$
we have $0\leq r<1$, $(e\cdot r)\leq n-1$, $(e\cdot r)\in N$. (3)
Conclusion 2. For distribution there are at most $(n-1)$ pieces of goods from among n markets.
Denote by:
$w=(w_k)$ – h – dimensional criterion of distribution of $(e\cdot r)$ number of goods,
$L=(\alpha_{kj})$ – zero – unity matrix of $w=(w_k)$ criterion value with respect to market $R=(R_j)$, $\alpha=(\alpha_j)=e\cdot L$.
We choose the dimension'h of criterion $w=(w_k)$ in such a way to get:
$\forall j_1,j_2\ j_1\neq j_2\Rightarrow\alpha_{j_1}\neq\alpha_{j_2}$, hence $h\geq n-1$.
Definition. We say that R_s precedes R_t and we write $R_s<R_t$ when $\alpha_s>\alpha_t$.
Relation $<\cdot$ is transitive in the set $\{R_j\}$ because
$R_s<\cdot R_t\wedge R_t<\cdot R_p\Rightarrow\alpha_s>\alpha_t\wedge\alpha_t>\alpha_p\Rightarrow\alpha_s>\alpha_p\Rightarrow R_s<\cdot R_p$.
For each pair of the elements $R_s\neq R_p\in\{R_j\}$ only one relation
$R_s<\cdot R_p, R_p<\cdot R_s$, occurs because $R_s<\cdot R_p\wedge R_p<\cdot R_s\Rightarrow\alpha_s>\alpha_p\wedge\alpha_p>\alpha_s\Rightarrow\alpha_s>\alpha_s$.
When the coordinates of market $R=(R_j)$ are arranged according to the relation $<\cdot$, it means$R_{j_1}<\cdot R_{j_2}<\cdot\ldots<\cdot R_{j_n}$ we define
$\delta^*=\bar{\delta}+\mu$ where (4)

$$\mu_{j_s}=\begin{cases}1, & \text{if } j_s\leq(e\cdot r),\\ 0, & \text{if } j_s>(e\cdot r).\end{cases} \qquad (5)$$

$(2)\wedge(4)\wedge(5)\Rightarrow\delta^*\in N$ and on the basis (4), (5) and (3) we have
$(e\cdot\delta^*)=(e\cdot(\bar{\delta}+\mu))=(e\cdot\bar{\delta})+(e\cdot r)=(e\cdot\vartheta\cdot\delta)=\vartheta\cdot(e\cdot\delta)=(e\cdot б)$.
Conclusion 3. For the demand δ^* the productivity of the factory F is fully used.
In my opinion it is the most natural procedure in the case where condition (1) is not satisfied. The reduced δ^* depends considenbly on the choice of criterion $w=(w_k)$ and hence it is not univocally defined. Similar problem exists in the problem of economic analysis, diet etc (see ref. [2]).

REFERENCES

[1] D. Gale, Theory of economic linear models, PWN, Warszawa 1969, (in Polish).
[2] S. Gass, Linear programming PWN, Warszawa 1976, (in Polish).

STRESZCZENIE

W artykule przedstawiono niesprzeczny algorytm dystrybucji znanego wektora podaży do wektora popytu według kryterium doboru $w=(w_k)$ przez zmianę popytu. Przy zmianie popytu na pierwszy plan wysunięto potrzebę zaspokojenia rynku.